A MEASURE OF JOB SATISFACTION BY MEANS OF FUZZY SET THEORY

Francesca De Battisti[1]

Department of Economics, Management and Quantitative Methods, University of Milano, Italy

Donata Marasini

Department of Economics, Quantitative Methods, and Business Strategies, University of Milano-Bicocca, Italy

Giovanna Nicolini

Evaluation Team of University of Milano-Bicocca, Italy

Received: 10th September 2013/ Accepted: 27th January 2014

Abstract*. In this paper fuzzy set theory is applied to measure satisfaction levels from various social aspects such as quality of life, employment conditions, or quality of a specific service, and so on. The theory is based on the construction of a membership function to a fuzzy set. In the literature some membership functions have already been suggested; the one proposed here seems to be more consistent with practical applications. The approach is used in a research project on "organisational well-being" and refers to a survey of employees of a public administration.*

Keywords*: Job satisfaction, Membership function, Fuzzy sets*

1. INTRODUCTION

The assessment of satisfaction from various social aspects such as quality of life, employment conditions or quality of a specific service requires an indicator measuring the level of subject satisfaction and latent variables that determine overall satisfaction.

In applying fuzzy set theory (Zadeh, 1965; Zimmermann, 2001, 2010) it is necessary to define a membership function (m.f.) of the fuzzy set being considered. For example, let X be an observed variable of continuous nature that has been discretized on an ordinal scale

---

[1] Francesca De Battisti, email: francesca.debattisti@unimi.it.

$$x_{(1)},\ldots,x_{(k)},\ldots,x_{(K)} \tag{1}$$

the set A of the pairs [$x_{(k)}$, $\mu_A(x_{(k)})$] is termed fuzzy, where $0 \le \mu_A\left(x_{(k)}\right) \le 1$ is a m.f. associated with $x_{(k)}$ such that

$$\begin{cases} \mu_A(x_{(k)}) = 0 & x_{(k)} \le x^* \\ \mu_A(x_{(k)}) > 0 & x^* < x_{(k)} < x^{**} \\ \mu_A(x_{(k)}) = 1 & x_{(k)} \ge x^{**} \end{cases} \tag{2}$$

where x^* and x^{**}, with $x^* < x^{**}$, are two terms from scale (1).

In this context, the m.f. value $\mu_A\left(x_{(k)}\right)$ is interpreted as the membership level in the virtual set A of satisfied people: 0 refers to complete dissatisfaction, 1 to complete satisfaction.

Fuzzy set theory has been applied in different frameworks such as measurement of poverty (Cerioli and Zani, 1990; Lemmi and Betti, 2006), well-being (Chiappero Martinetti, 2000; Baliamoune-Lutz, 2004), quality of life (Lazim and Osman, 2009), customer satisfaction for a service (Zani *et al.*, 2010; 2012) or satisfaction of graduates with the suitability of higher education for employment purposes (Crocetta and Del Vecchio, 2007).

The fuzzy set approach proposed in this paper is applied to the analysis of well-being at work, or job satisfaction, using a sample survey of employees of a public administration. Different m.f. are proposed: the first is related to a single subject for each item; the second, expanding the first, relates to n subjects; the third relates to a latent variable D with respect to n subjects; the last approach is again related to D but with respect to a single subject i.

In Section 2 the methodology of fuzzy sets is defined, in Section 3 satisfaction levels are interpreted in terms of fuzzy sets, in Section 4 some examples are shown and in Section 5 the results of a survey on employee satisfaction are presented. In Section 6 some conclusions are proposed.

2. FUZZY SETS

Referring to equation (2), the set of values $x_{(k)} > x^*$ are called as *support* of A and the set of values $x_{(k)} \ge x^{**}$ are defined as *core* of A. A can be termed as *normal* when it has at least an element of measure equal to 1, as in (2). A set can be called *singleton* if the support consists in a single element or as *empty* if the m.f. of all the elements is null.

The *union* of two fuzzy sets A and B, defined in the (1), is the fuzzy set $C = A \cup B$ such that

$$\mu_C\left(x_{(k)}\right) = \sigma\left[\mu_A\left(x_{(k)}\right), \mu_B\left(x_{(k)}\right)\right],$$

where σ is a function that takes values in the closed interval $[0,1]$, and it is symmetrical, associative, monotone, with neutral element 0.

We considere the following function:

$$\sigma\left[\mu_A(x_{(k)}), \mu_B(x_{(k)})\right] = \max\left[\mu_A(x_{(k)}), \mu_B(x_{(k)})\right]. \qquad (3)$$

The fuzzy set $D = A \cap B$ is termed as *intersection* such that

$$\mu_D\left(x_{(k)}\right) = \tau\left[\mu_A\left(x_{(k)}\right), \mu_B\left(x_{(k)}\right)\right]$$

with τ function which takes values in the closed interval $[0,1]$, and is symmetrical, associative, monotone, with neutral element 1. We chose the following two functions:

$$\begin{aligned} &\tau\left[\mu_A(x_{(k)}), \mu_B(x_{(k)})\right] = \min\left[\mu_A(x_{(k)}), \mu_B(x_{(k)})\right] \\ &\tau\left[\mu_A(x_{(k)}), \mu_B(x_{(k)})\right] = \mu_A(x_{(k)})\mu_B(x_{(k)}). \end{aligned} \qquad (4)$$

A** is a fuzzy set with S components A_s (s=1,2,…,S), each of which is a fuzzy set defined on the scale (1). ***B***** is another fuzzy set with T components B_t (t=1,2,…,T), each of which is defined on scale (1). ***A***** and ***B***** can be interpreted as matrices of measures with dimension S x K and K x T respectively.

Considering the S x T pairs (A_s, B_t) ($A_s \in \boldsymbol{A}^*$, $B_t \in \boldsymbol{B}^*$), we can associate the measure

$$\mu_{A^* \circ B^*} = max\left\{min\left[{}_{A^*}\mu_{A_s}\left(x_{(k)}\right),\ {}_{B^*}\mu_{B_t}\left(x_{(k)}\right)\right]\right\} \qquad (5)$$

where ${}_{A^*}\mu_{A_s}\left(x_{(k)}\right)$ denotes the measure associated with the value $x_{(k)}$ relating to the variable A_s of ***A*****. A similar meaning is associated with ${}_{B^*}\mu_{B_t}\left(x_{(k)}\right)$. The equation (5) is termed as *relation* or *composition* between ***A**** and ***B**** and can be seen as the result of the product of matrix ***A**** and the transposed ***B****, where the product of two generic measures is substituted by the second of the (4) and the sum by the (3).

A more generic definition of a fuzzy set than the one presented in (2) can be given. This definition is not necessarily related to a normal set. Specifically, if $x_1,\dots,x_k,\dots,x_K$ are K values with m.f. $\mu_A(x_k)$, the pairs set

$$\left[x_k, \mu_A(x_k)\right] \tag{6}$$

describes a fuzzy set A ($0 \leq \mu_A(x_k) \leq 1$).

3. FUZZY SETS AND SATISFACTION LEVELS

Let us consider n subjects, expressing their satisfaction level with a variable X_s of a generic dimension D. Considering the ordinal scale (1), we can associate a single subject i with the m.f.

$$\mu(i_s) = \frac{x_{(k)} - x^*}{x^{**} - x^*} \tag{7}$$

which is considered as a satisfaction measure in X_s if the response of i to the survey questionnaire is $x_{(k)}$ $\left(x^* \leq x_{(k)} \leq x^{**}\right)$.

With subject i, the associated fuzzy G_{is} is

$$\left[x_{(k)}, \mu(i_s) = \frac{x_{(k)} - x^*}{x^{**} - x^*}\right]$$

which coincides with a singleton if $x_{(k)} > x^*$ and with an empty fuzzy set if $x_{(k)} \leq x^*$. If all the values $x_{(k)} > x^*$ have at least a correspondent among the n, we have

$$G_s = \bigcup_{i=1}^{n} G_{is}.$$

If we denote the relative frequencies by $f_s\left(x_{(k)}\right)$ and the cumulative ones by

$$F_s\left(x_{(k)}\right)=\sum_{x_{(j)}=x_{(1)}}^{x_{(k)}} f_s\left(x_{(j)}\right) \tag{8}$$

leaving constant support and core (2), $x_{(k)}$ and equation (8) form a fuzzy set F_s.

The set $A_s = G_s \cap F_s$ with

$$\mu_{A_s}\left(x_{(k)}\right)=\frac{x_{(k)}-x^*}{x^{**}-x^*}F_s\left(x_{(k)}\right) \tag{9}$$

where the second part of equation (4) has been used, is a fuzzy set, with support and core (2), whose measures show the membership of all *n* subjects to the group of people satisfied with X_s.

Equation (9) can be used: a) to assign a m.f. to the variable X_s by the *n* subjects or the single subject *i*, considering as a virtual set the latent variable *D*, b) to assign a satisfaction measure relating *D* the subject *i,* and c) to compare pairs of latent variables.

In case a), first of all the *K* measures in (9) have to be synthesised to a single measure considered as the "weight" to be given to X_s. Putting

$$\mu_s^* = \sum_{x_{(k)}=x_{(1)}}^{x_{(K)}} \mu_{A_s}(x_{(k)})$$

and setting the measure

$$\mu_s = 1-\frac{\mu_s^* - \mu_{\min}^*}{\mu_{\max}^* - \mu_{\min}^*}$$

where we obtain $\mu_{\min}^*$ when all the relative frequencies are associated with value $x_{(K)}$ and $\mu_{\max}^*$ when all the frequencies are associated with one of the values $x_{(k)} \leq x^*$, we consider the following normalised version

$$\tilde{\mu}_s = \frac{\mu_s}{\sum_{s=1}^{S} \mu_s}. \tag{10}$$

If the values (1) coincide with the first K integer numbers, we have

$$\mu^*_{\min} = 1$$

$$\mu^*_{\max} = \frac{x^{**} - x^{*} + 1}{2} + K - x^{**}.$$

Let us consider S measurable variables that together compose the dimension D, equation (10) can be seen as a m.f. of X_s and can be interpreted as the level of membership of X_s in D, assigned by the n subjects.

Considering now the subject i, we can see that the pairs

$$[X_s, \tilde{\mu}_s] \tag{11}$$

produce a fuzzy set according to equation (6), and the same holds for the pairs

$$[X_s, \mu(i_s)] \tag{12}$$

where $\mu(i_s)$ is defined by equation (7). The intersection between the two fuzzy sets, for the second part of equation (4), leads to

$$\lfloor X_s, \tilde{\mu}_s \mu(i_s) \rfloor \tag{13}$$

where the second component identifies an m.f. expressing the level of membership of X_s in D given by the subject i.

In regard to case b), the synthesis of the measures given by equation (13), that is

$$\mu(i) = \sum_{s=1}^{S} \tilde{\mu}_s \mu(i_s) \tag{14}$$

can be seen as measure of satisfaction relating to the latent variable D for subject i.

In case c), to compare two latent variables we can proceed as follows: let S be the observed variables of a latent variable D, each of which describes a fuzzy set relating to scale (1). If we do not consider the core, the S variables produce a SxH matrix of measures, where $H < K$ are values of scale (1), and the m.f. of

$x_{(k)}$ relating to X_s, now indicated with ${}_D\,\mu_{X_s}\left(x_{(k)}\right)$, coincides with the right side of equation (9). Let *D'* be another latent variable of *T* dimensions which, with the previous limitations, is presented as an *H* x *T* matrix. The relation between the two matrices can be seen as the "product", as already stated in the previous section.

In other terms, the measure

$$\mu_{D\circ D'}\left(X_s, X'_t\right) = \max\left\{\min\left[{}_D\mu_{X_s}\left(x_{(k)}\right), {}_{D'}\,\mu_{X_t'}\left(x_{(k)}\right)\right]\right\} \qquad (15)$$

is associated with each pair $\left(X_s, X_t'\right)$, according to equation (5).

If the values given to the pair $\left(X_s, X_t'\right)$ coincide with the m.f. of *D*, we can say that *D* has a higher level of satisfaction compared to *D*.

If, instead, the m.f. of the matrix are some of *D* and some of *D*', the comparison of the two dimensions is difficult. This will be clarified in the following section. It is important to notice that for the structure of equation (15) the core has been cancelled because it would always produce a unitary matrix.

4. EXAMPLES

Let us consider the two following situations related to the variables $A_1 = X_1$ and $A_2 = X_2$ of dimension *D*:

Table 1: Example 1

$x_{(k)}$	$f_{A_1}\left(x_{(k)}\right)$	$\mu_{A_1}\left(x_{(k)}\right)$
1	0.40	0.00
2	0.40	0.20
3	0.20	0.50
4	0.00	0.75
5	0.00	1.00

Table 2: Example 1

$x_{(k)}$	$f_{A_2}\left(x_{(k)}\right)$	$\mu_{A_2}\left(x_{(k)}\right)$
1	0.10	0.00
2	0.30	0.10
3	0.30	0.35
4	0.30	0.75
5	0.00	1.00

where the last two columns, according to equation (9), are the measures associated with scale (1). Using equation (10) leads to m.f. $\tilde{\mu}_1 = 0.13$ for X_1 and $\tilde{\mu}_2 = 0.87$ for X_2, that is a higher level of satisfaction for *D* expressed by X_2 compared to X_1. If the subject *i* has answered, for example, 3 for X_1 and 2 for X_2, according to equation (7) we have for the first 0.5 and 0.25 for the second, so that using equation (14) allows us to give to *i* a measure of satisfaction equal to $\mu(i) = 0.25$.

Comparing the two m.f. shown in the last two columns of Tables 1 and 2 the level of satisfaction for X_1 seems to be higher than the one for X_2 since the m.f. of the first is higher or equal to the second. As a matter of fact, comparing the two, the interpretation follows a different logic since the m.f. depends on cumulative frequencies. It follows that the rule which associates a higher level of satisfaction to the m.f. which are close to unity has an opposite meaning. Anyway, the comparison of items must not be made among the m.f. $\mu_{A_2}\left(x_{(k)}\right)$ depending on $x_{(k)}$ (k = 1,…,K) but among the m.f. $\tilde{\mu}_s$ depending on s (s = 1,…,S).

Let us consider now a second dimension *D'* of two variables X_1' and X_2', with:

Table 3: Example 1

$x_{(k)}$	$f_{A_1}\left(x_{(k)}\right)$	$\mu_{A_1}\left(x_{(k)}\right)$
1	0.00	0.00
2	0.10	0.03
3	0.40	0.25
4	0.40	0.68
5	0.10	1.00

Table 4: Example 1

$x_{(k)}$	$f_{A_2}\left(x_{(k)}\right)$	$f_{A_2}\left(x_{(k)}\right)$
1	0.00	0.00
2	0.00	0.00
3	0.20	0.01
4	0.50	0.53
5	0.30	1.00

According to the (10) the measures are $\tilde{\mu}_1' = 0.38$ and $\tilde{\mu}_2' = 0.62$; if the subject *i* has again answered 3 for X_1' and 2 for X_2', it is given a measure equal to $\mu'(i) = 0.35$, to the same subject.

The two matrices are instead:

	2	3	4
X_1	0.20	0.50	0.75
X_2	0.10	0.30	0.75

	2	3	4
X_1'	0.03	0.25	0.68
X_2'	0 .00	0.01	0.53

and their relation according to the (15) is:

	X_1'	X_2'
X_1	0.68	0.53
X_2	0.68	0.53

Since the values of the matrix are the m.f. of the observed variables of *D'* we can say that *D'* has a higher level of satisfaction than *D*.

5. APPLICATION

The methodology presented in the previous sections has been applied to analyse the data from a survey conducted on the *organisational well-being* of employees of a public administration in Italy.

The organisational well-being is becoming more and more important since it is related to psychological illnesses that can derive from a lack of well-being in the working environment. These illnesses can represent a social cost as they have to be diagnosed and treated. In this context, satisfaction is taken in consideration with respect to the organisational well-being.

The questionnaire examines ten dimensions or latent variables. For this application we propose the following three dimensions which we consider most significant: D_1, D_2 and D_3, with: D_1 = relationships with colleagues, D_2 = organizational equity and D_3 = relationships with supervisors. For the last one. A detailed elaboration is presented. Items considered for each dimension are the following:

- ${}_1X_1$ = Is there partnership with colleagues?
- ${}_1X_2$ = In the working group who has information makes it available to everyone?
- ${}_1X_3$ = In the working group everyone is committed to achieve results?
- ${}_1X_4$ = Among colleagues we listen and we try to meet each other's needs?
- ${}_2X_1$ = Does the University offer career opportunities for everyone?

- ${}_2X_2$ = Are commitment at work and personal initiative appreciated (with rewards, commendations, etc.)?
- ${}_2X_3$ = Are there opportunities for updating and professional development?
- ${}_2X_4$ = Are economic incentives distributed on the basis of the effectiveness of performance?
- ${}_3X_1$ = Do the supervisors treat their employees fairly?
- ${}_3X_2$ = Is the supervisors' behaviour in agreement with the stated objectives?
- ${}_3X_3$ = Do the supervisors want to be informed about the problems and the difficulties their employees have at work?
- ${}_3X_4$ = Do the supervisors involve their employees in decisions related to their own work?
- ${}_3X_5$ = Are the evaluation criteria fair and clear?

The questionnaire was submitted to 715 employees belonging to four management levels (except the executives) and pertaining to all the sectors of the organisation. Only 61% of the employees completed the questionnaires; however, only 295 questionnaires with complete data have been taken into consideration. The modalities $x_{(k)}$ of each observed variable are four, such as: $x_{(1)} = 1$ definitely no, $x_{(2)} = 2$ partially no, $x_{(3)} = 3$ partially yes, $x_{(4)} = 4$ definitely yes. The sum of the frequencies of the first two items expresses how many employees are dissatisfied; and, the sum of the frequencies of the last two items measures how many employees are satisfied. Such frequency is shown in Table 5 for the items of each dimension.

Table 5: Frequencies related to the satisfied of the items of each dimension

D_1	D_2	D_3
${}_1X_1$ 0.80	${}_2X_1$ 0.15	${}_3X_1$ 0.66
${}_1X_2$ 0.72	${}_2X_2$ 0.21	${}_3X_2$ 0.63
${}_1X_3$ 0.76	${}_2X_3$ 0.32	${}_3X_3$ 0.65
${}_1X_4$ 0.76	${}_2X_4$ 0.22	${}_3X_4$ 0.44
		${}_3X_5$ 0.42

In regard to dimension D_2 (*organisational equity*) we can see the lowest frequencies of people satisfied, while in regard to D_1 (*relationships with colleagues*) we can see the highest frequencies of people satisfied.

In regard to dimension D_3 and applying the methodology presented in Section 3, the aims are:

a) to define the m.f. of each item of a dimension and make a comparison;
b) to define a measure of satisfaction for each subject relating to the

considered dimension;

c) to compare pairs of dimensions.

To do this we have to refer to the following functions:

- ✓ $\mu(i_s)$, defined with the (7), relating to subject *i*.
- ✓ $\mu_{A_s}(x_{(k)})$, defined with the (9), producing the m.f. of the 295 subjects relating to the fuzzy A_s, that is the m.f. of all the subjects expressing a value in the scale of each item of the dimension. In regard to D_3 such functions are shown in Table 6.

Table 6: m.f. $\mu_{A_s}(x_{(k)})$ of 295 subjects for each value of the scale of the items of dimension D_3

$x_{(k)}$	1	2	3	4
${}_3X_1 = A_1$	0	0.11	0.51	1
${}_3X_2 = A_2$	0	0.12	0.55	1
${}_3X_3 = A_3$	0	0.12	0.46	1
${}_3X_4 = A_4$	0	0.19	0.61	1
${}_3X_5 = A_5$	0	0.19	0.58	1

The case a) is satisfied by $\tilde{\mu}_s$ which is m.f. relating to fuzzy (11). The function shows the level of satisfaction for each item of the dimension D_3 expressed by the whole group of respondents (Table 7).

Table 7: m.f. $\tilde{\mu}_s$ for each item of dimension D_3

${}_3X_1$	${}_3X_2$	${}_3X_3$	${}_3X_4$	${}_3X_5$
0.24	0.21	0.27	0.13	0.14

The highest level of membership, given by the 295 subjects to the group of people satisfied for the dimension D_3, is equal to 0.27 and in correspondence to item ${}_3X_3$. Despite being a relatively low value, however, we can see in Table 6 that the lowest m.f. value is associated with this variable and in correspondence to the scale value 3; this confirms what stated in Section 4 as regards Examples 1 and 2.

In regard to case b) it has been used $\tilde{\mu}_s\mu(i_s$ expressing the level of membership of an item to the dimension given by a single subject. Adding these functions for all the items of one dimension, according to the (14), we have the measure of satisfaction of each subject $\mu(i)$ for one dimension. The frequency distribution of these measures for D_3 is shown in Table 8.

Table 8: Frequency distribution of values $\mu(i)$ for dimension D_3

Classes	0-0.1	0.1-0.2	0.2-0.3	0.3-0.4	0.4-0.5	0.5-0.6
n_i	18	15	23	31	44	23
%	6.1	5.1	7.8	10.5	14.9	7.8
Classes	0.6-0.7	0.7-0.8	0.8-0.9	0.9-1	Amount	
n_i	53	34	19	35	295	
%	18	11.5	6.4	11.9	100	

In regard to case c) the dimensions can be compared according to the (15). Tables 9 and 10 show the comparison of dimensions D_3 - D_1 and D_3 - D_2.

Table 9: Comparison of dimensions D_3 and D_1

	${}_3X_1$	${}_3X_2$	${}_3X_3$	${}_3X_4$	${}_3X_5$
${}_1X_1$	0.41	0.41	0.41	0.41	0.41
${}_1X_2$	0.46	0.46	0.46	0.46	0.46
${}_1X_3$	0.46	0.46	0.46	0.46	0.46
${}_1X_4$	0.49	0.49	0.49	0.49	0.49

Table 10: Comparison of dimensions D_3 and D_2

	${}_3X_1$	${}_3X_2$	${}_3X_3$	${}_3X_4$	${}_3X_5$
${}_2X_1$	0.51	0.55	0.46	0.61	0.58
${}_2X_2$	0.51	0.55	0.46	0.61	0.58
${}_2X_3$	0.51	0.55	0.46	0.61	0.58
${}_2X_4$	0.51	0.55	0.46	0.61	0.58

The values shown in Table 9 coincide with the m.f. of dimension D_1, therefore being the best one in terms of satisfaction, relating to dimension D_3; in Table 10 there are the m.f. values of dimension D_3, which is therefore the best one relating to D_2.

Considering now the possible comparisons in pairs between the analysed dimensions, we can say that dimension D_1 always takes values of m.f. lower than dimensions D_2 and D_3, therefore being the best one. Then we have a dimension ranking, D_1 - D_3 - D_2, which is coherent with what shown in Table 5, where D_1 is the best one and D_2 is the worst one.

6. CONCLUSIONS

In this paper the levels of satisfaction and a measure of satisfaction have been defined with reference to fuzzy set theory. The paper suggests m.f. and their synthesis that seem to fit satisfactorily to real cases. Furthermore the elaboration of the proposed m.f. allows to examine several aspects and relations between latent and observed variables. Specifically: for each observed variable a m.f. has been defined for all the subjects related to the satisfaction of the same variable; moreover, for each observed variable, an m.f. relating to the satisfaction of the correspondent dimension has been defined. For each pairs of dimensions a comparison of the m.f. has been proposed from which a ranking on the level of satisfaction for each dimension can be made. Finally, for each subject has been defined an m.f. relating to the satisfaction of an observed variable belonging to a dimension and a measure of satisfaction for each dimension. Such measures can be synthesized through a suitable function, such as the arithmetic mean or a weighted mean, obtaining an overall satisfaction measure for each subject. The proposed methodology has been applied to a survey on organisational well-being of employees of a public administration, obtaining interesting results both on the extent of satisfaction for each variable, which allowed the construction of a ranking, and with respect to pairs of variables, which allowed a comparison between them.

REFERENCES

Baliamoune-Lutz, M. (2004). *On the Measurement of Human Well-Being: Fuzzy Set Theory and Sen's Capability Approach*, UNU-WIDER, Helksinki.

Chiappero Martinetti, E. (2000). A multidimensional assessment of well-being based on Sen's functioning approach. In *Rivista Internazionale di Scienze Sociali*, 108(2): 207-239.

Cerioli, A., and Zani, S. (1990). A fuzzy approach to the measurement of poverty. In: C. Dagum, M. Zenga, editors., *Income and Wealth Distribution, Inequality and Poverty*, Springer, Berlin: 272-284.

Crocetta, C., and Delvecchio, G. (2007). A fuzzy measure of satisfaction for university education as a key for employment. In L. Fabbris, editor, *Effectiveness of University Education in Italy*, Phisica-Verlag, Heidelberg: 11-27.

Lemmi, A., and Betti, G. (2006). *Fuzzy set spproach to multidimensional poverty measurement*, Springer, New York.

Lazim, M.A., and Osman, M.T.A. (2009). A new Malaysian quality of life index based on fuzzy sets and hierarchical needs. In *Social Indicators Research*, 94(3): 499-508.

Zadeh, L.A. (1965). Fuzzy sets. In *Information and Control*, 8: 338-353.

Zani, S., Milioli, A., and Morlini, I. (2010). Fuzzy composite indicators: An application for measuring customer satisfaction. In *Proceedings of the Meeting of the Italian Statistical Society*, Padua [CD].

Zani, S., Milioli, A., and Morlini, I. (2012). Fuzzy methods and satisfaction indices. In R.S. Kenett, S. Salini editors, *Modern Analysis of Customer Surveys*, Wiley, New York, 439-456.

Zimmermann, H.J. (2001). *Fuzzy sets theory and its applications*, Kluver Academic Publishers, USA.

Zimmermann, H.J. (2010). Fuzzy sets theory. In *Wiley Interdisciplinary Reviews: Computational Statistics*, 2(3): 317-332.