

MEASURING CLUTCH PERFORMANCE IN PROFESSIONAL TENNIS

Stephanie Ann Kovalchik¹

Game Insight Group, Tennis Australia, National Tennis Centre, Melbourne Park

Machar Reid

Game Insight Group, Tennis Australia, National Tennis Centre, Melbourne Park

Abstract. *Not all points in tennis are of equal importance, yet current performance evaluation ignores the variable importance of points in a match. This paper introduces ‘clutch averaging’, a general method for evaluating player performance on important points. Clutch averages are statistical summaries that are weighted by the probabilistic importance of points. Using point-level data of men’s and women’s Grand Slam matches, we compare the discriminatory ability and predictive accuracy of clutch averaging and simple averages on a set of 11 serve and return performance measures. We find that clutch averages generally improved prediction accuracy and discrimination compared to simple averages. This general and easy-to-use method for accounting for importance in summaries of tennis match performance will be a useful tool for understanding match outcomes and the impact of pressure on performance in elite tennis.*

Keywords: *Discriminant analysis; Elite athletes; Performance evaluation; Sports statistics*

1. INTRODUCTION

In sport, *clutch* performance refers to the ability of an athlete to excel when it matters most. Clutch ability and related psychological phenomena like *choking* are a fascination of fans and sports commentators. Whole texts in sport are devoted to clutch performance and the idea that performing well under pressure is a defining characteristic of champions (Afremow, 2015). Although empirical evidence of clutch ability across sports is mixed (Hibbs, 2010; Solomonov et al., 2015), it is well-established that athlete performance is often affected by the amount of pressure experienced during competition (Wang et al., 2003).

¹ Corresponding author: S. A. Kovalchik, e-mail: skovalchik@tennis.com.au

The role of clutch ability is of particular interest in individual sports, like tennis, where player psychology is thought to have a greater influence on game outcomes than in team sports (Hill and Shaw, 2013). Indeed, tennis is frequently referred to as ‘the mental game’, a label that presumes that player mentality is critical to success in the sport (Weinberg, 2013). There is a growing body of empirical work to support this popular notion. The seminal work of Klaassen and Magnus (2001) established the systematic variation in tennis player performance with game pressure and momentum. Focusing specifically on break points as their measure of pressure, Knight and O’Donoghue (2012) found that receiving players have a higher likelihood of winning break points than less pressured points. González-Díaz et al. (2012) used similar variation under pressure to define player ‘critical ability’ and found notable variation in this ability among elite players.

More recently, Kovalchik and Ingram (2016) used performance variation across a range of game situations with varying pressure to identify mental profiles in tennis, research that revealed a distinct signature in responses under pressure among the most accomplished male players.

Although there is substantial evidence that, when it comes to performance, tennis players don’t treat every point the same, conventional statistics of tennis performance ignore these effects. For many decades, the standard match analysis provided by tennis broadcasters and analysts has been the comparison of percentage statistics (e.g. service points won) between the winner and loser of a match. These percentages treat every point in a tennis match as equally important. However, this method is inconsistent with the reality of the sport, where points are *not* equally important to the match outcome, and makes for perplexing analysis. For example, the winner in 1 of every 20 matches in professional tennis wins fewer points than the loser (Wright et al., 2013). Without a method to explain paradoxical results like these, commentators and analysts can struggle to articulate the factors that determined a match win and risk losing credibility with players and fans (Goldstein, 1979; Keene and Cummins, 2009).

The present work offers a solution to this problem of tennis commentary and performance analysis by providing an alternative approach to match performance summarizing. The general summary method we propose is named *clutch averaging*. It’s primary feature is that it incorporates point importance in the evaluation of a particular match skill of interest and weighs performance on the most important points in a match more heavily. The purpose of this method is to provide an easy-to-use and reliable tool for performance analysis in tennis that emphasizes clutch ability and provides direct insight into the mental side of the game from in-competition data.

2. METHOD

To quantify clutch, we need to identify high-pressure situations in a match and focus on how players perform in those situations. This process is complicated by the different types of pressure that a player can experience. For example, a player might feel pressure with the threat of a loss when behind in the score but equally feel pressure when in a position to close out a set or match. Although quite different situations, both are examples where the outcomes of particular points can have a large influence on the outcome of the match.

The traditional way that researchers in tennis have identified such critical or ‘big points’ is with *importance*. Let $M(a, b)$ represent the probability that the player who is currently serving wins the match given that the score at the start of the point for the server (e.g. 30-all, at 3-2 in the deciding set) is a and the score for the returner is b . A technical definition of importance was developed by (Morris, 1977) and can be written as follows

$$Imp(a, b) = M(a + 1, b) - M(a, b + 1) \quad (1)$$

where we use $a + 1$ to denote the change in score if the current point is won by the server. Thus, this definition says that the most important points are those that result in the greatest change in winning a match when the point is won compared to when the point is lost.

We illustrate the calculation of point importance in Table 1. This table shows ten of the most important points in a best-of-five set match with a tiebreak deciding each set, if necessary. In these calculations it is assumed that both servers win 65% of points on serve, which is the average on the ATP and represents the importance for equally matched opponents. As has been discussed previously, four of the important points are the final points in a match-deciding tiebreak, where the importance is 50% (O’Donoghue, 2001). The other points are set-deciding situations or game-deciding situations, outside of a tiebreak, in the final set of a match. These values can be contrasted with the first point of the match, a relatively unimportant point for the match outcome, which has an importance of 3%.

To evaluate clutch performance we introduce a *clutch averaging method*. This averaging weighs events of interest by the importance of the point during which the event occurred. The importance weight is defined as follows which means that performance in the score situation (a, b) is weighted by the importance of all points that have greater importance than θ . For all other situations, the weight is zero and the performance on those points do not factor into the average.

Tab. 1: Ten of the most important points in a best-of-5 tennis match.

Server Point	Score-Returner Game	Score Set	Wins Match	Probability Loses Match	Importance
0-0	0-0	0-0	51%	52%	3% (Reference)
5-6	6-6	2-2	50%	100%	50%
5-5	6-6	2-2	83%	68%	50%
6-6	6-6	2-2	83%	68%	50%
6-5	6-6	2-2	100%	50%	50%
30-40	4-5	2-2	39%	100%	39%
30-40	4-4	2-2	47%	91%	39%
30-40	5-5	2-2	47%	91%	39%
30-40	5-4	2-2	89%	50%	39%
30-40	6-5	2-2	89%	50%	39%
4-5	6-6	2-2	50%	89%	39%

$$W(a, b) = \begin{cases} Imp(a, b) & Imp(a, b) > 0 \\ 0 & \text{Otherwise} \end{cases} \quad (2)$$

With these weights defined, we can now introduce a *clutch average*. Let $Y_i(a, b)$ be an indicator of whether the event of interest (e.g. first serve in, second serve point won, etc.) took place and let $Imp_Y(a, b)$ be the indicator for whether the event could have occurred for the i th player at the point (a, b) . The clutch average for this skill is which is the weighted average across all points in the match, normalised to the total importance of the points played. Standard reporting of tennis performance statistics uses simple averaging, which sets $W(a, b) = 1$ for all points. From the expression above, we see that *simple averaging* implicitly assumes that all points are equally important.

3. ILLUSTRATION

One of the most surprising matches at the 2016 Australian Open was the five-set Round of 16 match between Novak Djokovic and Gilles Simon. As the No. 1 seed, Djokovic was the favorite to win the tournament, which made the up-and-down match against Simon a real surprise to fans and commentators.

When fans, players and coaches turn to match statistics, they often are looking for an explanation for why a match progresses in the way that it does. Points won on serve is won of the key statistics that is looked to when trying to understand how sets and matches are won. However, when summaries of service points won ignore point importance, they can give a misleading impression of a player's win expectations.

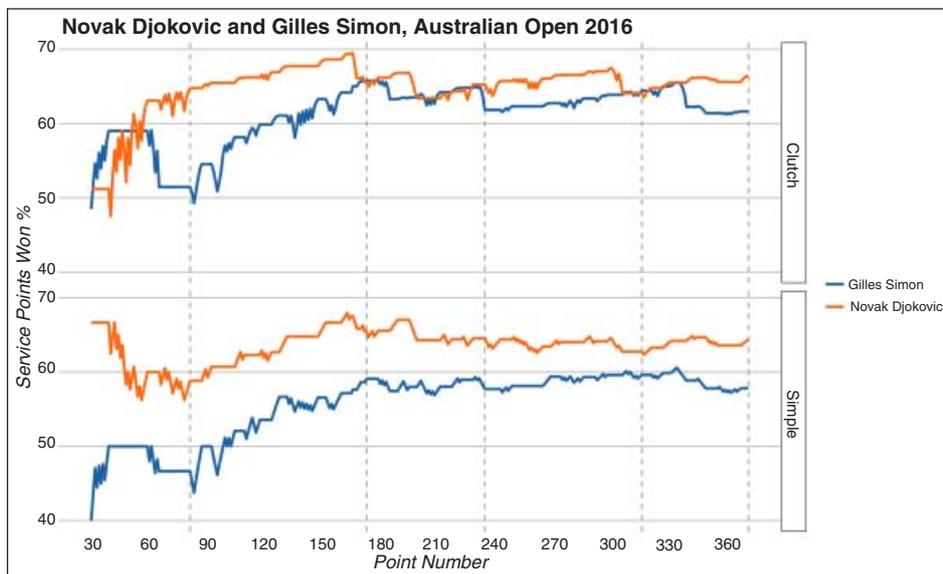


Fig. 1: Clutch and simple averages for service points won for the 2016 Australian Open match between Novak Djokovic and Gilles Simon from the 30th point of the match to the final point

We can see this illustrated in Figure 1 when we contrast the clutch service points won to the simple average of service points won. According to the cumulative simple average, Djokovic had a large lead on serve throughout the match, despite losing the second and fourth set of the match. In contrast, the cumulative clutch serve performance, with $\theta=0$, shows a number of reversals in which player was leading this performance measure.

Several reversals at the end of sets (shown by the dashed vertical lines) not only align with the winner of the set, they also highlight turning points in serve performance. Although only a single match and single performance statistic, this illustration shows the potential for clutch averaging to better capture the dynamics of performance and do so in a way that is more predictive of the outcome.

4. PERFORMANCE

4.1. DATA

The performance of the clutch averaging method was evaluated in a sample of 305 men's and 296 women's Grand Slam matches from 2011, which were obtained from a publicly available dataset on the www.tennisabstract.com website. The dataset included point-by-point scores and indicators for a variety of events

including the 11 events listed in Table 2. The event measures are categorized as point, return, or service events according to the type of points in which the event can occur. The summaries of the frequencies of each event show that the percentage of total points won, service points won, break points created and break points converted have some of the largest average mean differences between the players who win and players who lose matches.

Tab. 2: Summary of Study Sample and Mean Percentage (IQR) of Skill Measures Evaluated

Event Measures	ATP		WTA	
	Winner	Loser	Winner	Loser
Matches	305	296		
<i>Point</i>				
Points Won	55.7 (4.3)	44.3 (4.3)	57.0 (6.2)	43.0 (6.2)
Winners	0.2 (0.1)	0.1 (0.1)	0.3 (0.2)	0.2 (0.1)
Unforced Errors	0.1 (0.1)	0.2 (0.1)	0.3 (0.2)	0.3 (0.2)
<i>Serve</i>				
Service Points Won	71.5 (8.1)	59.4 (7.8)	66.4 (10.0)	52.3 (10.2)
Aces	10.0 (8.3)	5.9 (5.6)	4.8 (5.3)	2.5 (3.6)
Double Faults	2.8 (2.9)	3.9 (3.5)	4.7 (4.8)	6.1 (5.2)
First Serve In	64.5 (9.3)	62.5 (8.8)	66.4 (10.0)	64.9 (10.9)
First Serve Won	77.1 (9.6)	65.8 (10.0)	70.7 (11.6)	56.5 (10.7)
Second Serve Won	61.4 (13.0)	49.0 (11.5)	58.2 (16.7)	45.1 (14.2)
<i>Return</i>				
Break Points Created	11.5 (5.4)	5.9 (5.0)	15.8 (7.5)	8.4 (6.10)
Break Points Won	49.3 (21.5)	33.1 (35.7)	54.8 (22.9)	39.7 (27.8)

4.2. ANALYSIS

Two attributes of the clutch average performance were evaluated: discrimination and prediction accuracy. Discrimination was evaluated by looking at the mean difference in the end-of-match averages of the winner and loser of the match. In this context, a more discriminating metric is one that consistently shows a greater separation between the performance of the winner and loser of a match.

Prediction accuracy was measured by calculating the difference in the event statistic at the end of the first set and determining how predictive a positive differential was of the match winner using overall accuracy as the predictive measure.

We also measured the added value of the first set clutch differential using a logistic regression model adjusted for the difference in the simple average. Thus, the effect observed would indicate the *additional* predictive value of the clutch

average after accounting for the predictive information contained in the difference in the simple averages. In these analyses, the order of the players in the difference statistic was randomly selected. Differences in the logistic regression analysis were also standardized by subtracting the mean and dividing by the standard deviation so that the magnitude of the effect could be directly compared across measures.

The performance evaluation was conducted for the 11 different events statistics listed in Table 2 and each for every value of θ in (0, 0.005, ..., 0.05). This range of thresholds were influenced by the distribution of importance in the data sample, where the 25th percentile and median for men's matches were 1% and 3% and for women's matches were 3% and 5%. Throughout, importance weights for the ATP were based on a match of equal opponents who win 65% of points on serve and for the WTA a match in which players win 57% of points on serve, the current averages for each tour. Both the discriminatory power and predictive ability were contrasted with the corresponding performance of simple averaging.

All analyses were performed in the R statistical programming language. Statistical inferences were based on the 95% confidence interval of the performance measures evaluated.

5. RESULTS

Differences between the clutch performance of winners and losers of matches were generally larger than the differences in the corresponding simple averages (Figure 2). Among the point measures, at $\theta=0$, the clutch average for the percentage of points won, winners and unforced errors were 3.7, 0.5 and -2.0 percentage points different from the corresponding simple differences for the ATP; for the WTA, the corresponding differences were 3.7, 0.7 and -1.5. Note that we expect positive differences for statistics that added to a winner's score, such as points won, and a negative difference for statistics that deduct from a winner's score, such as unforced errors.

At $\theta = 5\%$, the highest threshold for deciding which points would receive a non-zero weight in the clutch average, the differences were more stark. For the ATP, the differences compared to the simple averages in points won, winners and unforced errors were 8.8, 1.0 and -5.6; for the WTA, the differences were 9.3, 2.0 and -4.0.

For the service events considered, at the lowest valued θ condition of 0, the differences between the clutch and simple averages were 4.3, 1.2, 3.4, 4.7, 0.2 and -0.3 for total service points won, first serves in, first serves won, second serve wins, aces and double faults for ATP players. For women players, the corresponding differences were 3.8, 0.8, 3.5, 3.6, 0, and -0.4. At the highest value of θ , the men's differences generally increased, with the mean differences becoming 10.7, 1.6, 7.7,

7.0, 0.5, 0.4, and -1.3. For the women, the differences changed to 9.8, 0.9, 8.9, 6.2, 0.4, and -1.7. Thus, we observe greater differences in the expected direction with clutch averaging of service events with the exception of the rate of aces, where differences were comparable to simple averaging.

The creation of break points and break points converted also had notable improvement in discrimination with clutch averaging. With $\theta=0$, the men's break points created and break points won were 5.6 and 3.3 percentage points greater with the clutch average than the simple average. For women, the differences were 2.9 and 3.9. At $\theta=5\%$, the discrimination improvement for break points created increased to 10.0 and for break points won increased to 6.9 percentage points. For women, the corresponding improvements were 10.0 and 8.4.

The pattern of discrimination shows general improvement with an increasing value of θ , that is, a higher threshold for importance (Figure 2). However, as fewer points are included in the average with increasing θ this also comes with an increase in uncertainty in the differences between players.

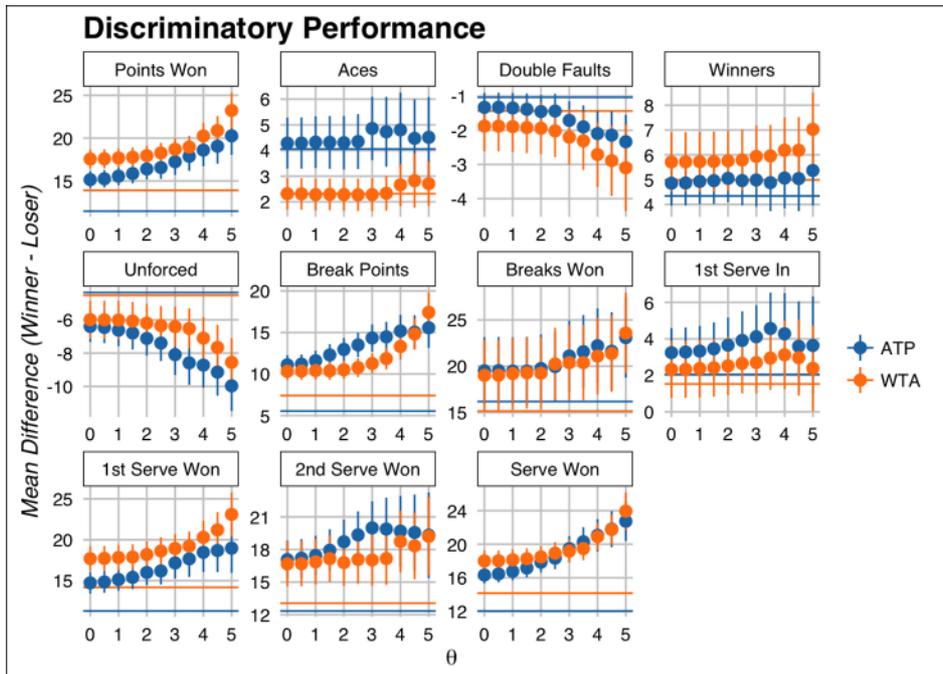


Fig. 2: Discrimination performance of performance measures showing the mean difference and 95% confidence intervals over a range of θ . Horizontal lines show the discrimination for simple averages.

The added predictive value of the clutch averages varied across measures and tour (Figure 3). As with the direction of the discrimination, we expect the odds ratio for winning would be above 1 for factors that add to a winner’s score and less than 1 for factors, like unforced errors, that deduct from a winner’s score. At the lowest threshold of $\theta = 0$, the events whose clutch averages added statistically significant value to predicting the match outcome at the end of the first set were total service points won (OR = 1.8, 95% CI = 1.0 - 3.7), first service points won (OR = 2.5, 95% CI = 1.4, 4.6), second service points won (OR = 1.6, 95% CI = 1.1 - 2.6), break points created (OR = 3.8, 95% CI = 1.8 - 8.6) and unforced errors (OR = 0.39, 95% CI = 0.2 - 0.7). Notably, break points won showed a negligible effect across the weight thresholds, which could be explained by the fact that most break points will have a similar level of importance and will behave more like a simple average than the other statistics.

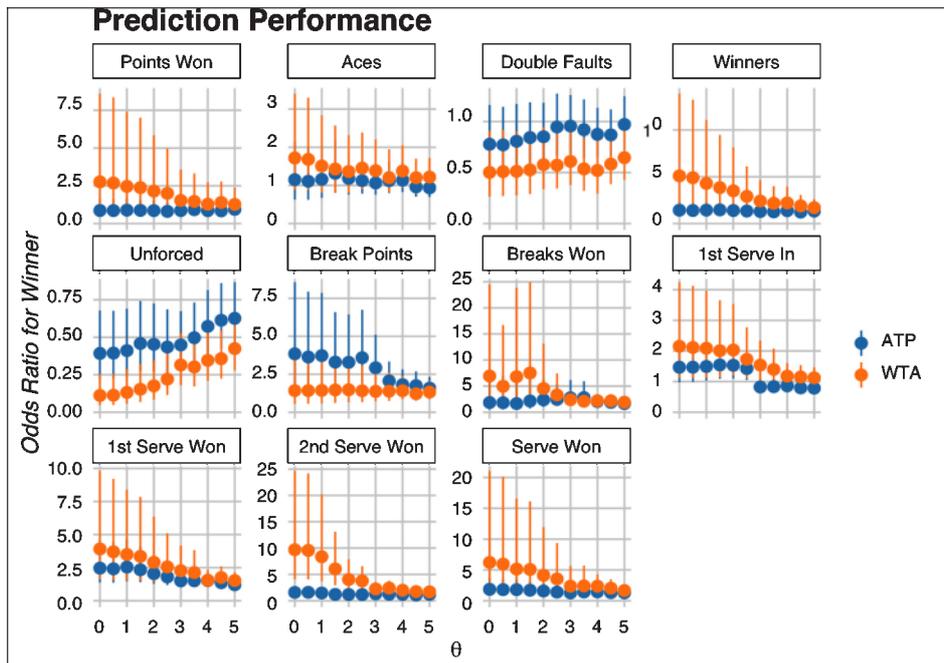


Fig. 3: Adjusted odds ratio of the relationship between the player clutch differences at the end of the first set, adjusted for the differences in the simple averages. All differences were standardized. Points show the adjusted odds ratio and 95% confidence intervals over a range of θ .

For women, the significant effects were service points won (OR = 6.2, 95% CI = 2.0 - 21.1), service points won (OR = 6.2, 95% CI = 2.0 - 21.1), first service points in (OR = 2.2, 95% CI = 1.1 - 4.2), first service points won (OR = 3.9, 95% CI = 1.6 - 9.8), second service points won (OR = 9.7, 95% CI = 4.1 - 24.7), break points won (OR = 6.9, 95% CI = 2.1 - 24.5), unforced errors (OR = 0.1, 95% CI = 0.1 - 0.3), and winners (OR = 5.0, 95% CI = 2.0 - 13.9).

Comparison of clutch averages in the first set of a match improved the accuracy of predictions of the match winner for 8 of the performance measures for men's matches. The highest observed improvement for each of these measures of 2 percentage points for the percentage of first serves in, 3 percentage points for percentage of first service points won, 2 percentage points for break points created, 2 percentage points for break points won, 1 percentage point for double faults, 1 percentage point for service points won, 6 percentage points for unforced errors, and 1 percentage points for winners (Figure 4). For women's match statistics, we observed even more improvement in prediction accuracy with the first set clutch differentials compared to simple averages with ten of the performance measures being more predictive with clutch averaging. The highest observed improvement was 3 percentage points for the percentage of first service points in, 3 percentage points for first service points won, 2 percentage points for second service points won, 3 percentage points for aces, 3 percentage points for break points created, 1 percentage point for double faults, 1 percentage point for total points won, 2 percentage points for service points won, 6 percentage points for unforced errors, and 2 percentage points for winners (Figure 4).

In general, the strength of the predictive effects diminished with increasing values of θ . This could be explained by the increased variance that results when using a higher importance threshold. We also observed stronger effects for the WTA for most measures compared to the men. We believe this is a consequence of the difference in match formats for the men's and women's Grand Slam matches. Since women play a best of 3 format and men play a best of 5, the information at the end of the first set would represent a higher percentage of the match information for women than men.

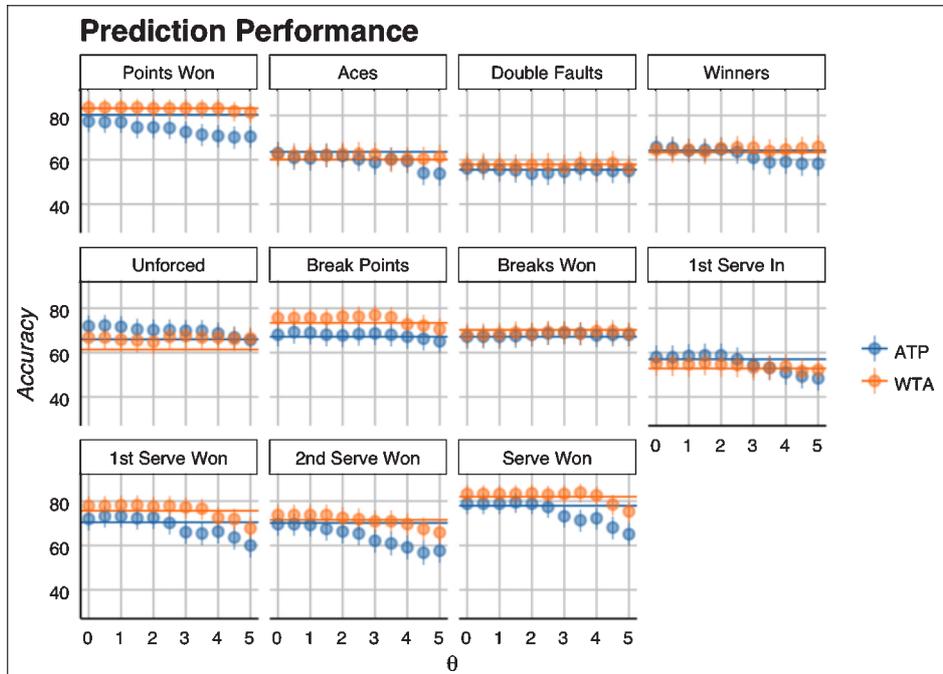


Fig. 4: Match prediction accuracy based on first set clutch differentials versus simple differentials (horizontal lines). Points show the overall accuracy and 95% confidence intervals over a range of θ .

6. DISCUSSION

This paper has introduced *clutch averaging*: a method for incorporating point importance into statistical summaries of tennis matches. This method up weights points with greater probabilistic importance and, in doing so, focuses the evaluation of player performance on the points that are most decisive for the match outcome. When we contrasted the discriminatory performance of clutch averages against simple averages, we found that clutch statistics were more strongly related to the match outcome and better able to differentiate the performance of the winners and losers of matches.

The use of clutch averaging represents an important advance in how analysts evaluate performance in tennis. Tennis broadcasters and commentators have ignored point importance for decades, leaving out a critical dimension of the sport in their performance evaluations. Similarly, while the increased availability of point-level data has spurred more detailed research on professional matchplay (Moss and O'Donoghue, 2015; Reid et al., 2016), this research continues to treat

all points as equally important, which raises concerns about the relevance of the results. Indeed the lack of context of many tennis statistics has attracted criticism (Wei et al., 2016) and calls for future performance research, particularly when combined with shot level information, to better account for the game context.

Clutch averaging is a type of weighted average that uses variable weights rather than the conventional equal weighting approach to account for score context. There are many types of variable weights one could consider. In this paper, we evaluated a family of weighting approaches that centered on the importance of points. We chose to focus on probabilistic importance because of its direct connection with match wins (Morris, 1977) and also because prior studies have shown systematic differences in how player perform on more important points compared to less important points (Klaassen and Magnus, 2001; Knight and O'Donoghue, 2012).

The family of importance weights we considered differed in the threshold for defining points of low importance that were assigned a weight of zero. Although any choice of threshold for defining low importance improved discriminatory ability overall, we found that a threshold of 3% importance for the men's game and

5% importance for the women's game were the levels where discriminatory ability peaked for most of the statistics considered. Because more points of lower importance receive a weight of zero with a higher threshold, this finding suggests that increasing the variance in the importance weights tends to improve discriminatory performance. An area for further research would be to examine other variable weighting approaches that could further improve on the performance of the clutch methodology.

One of the main reasons that the choice of weights matters is that player performance is affected by pressure. If players played all points the same, we would not observe any statistical differences in the discriminatory ability of averages with different choices of weights. This finding is consistent with prior studies that have shown that elite players do not play every point the same but tend to perform below their average on more important points (González-Díaz et al., 2012; Kovalchik and Ingram, 2016). Clutch averages provide a direct measure of how players perform under pressure. However, it remains unknown whether the pressure reflected by the probabilistic importance matches player perception of pressure. Since we would expect variation in performance to be most closely linked to player perceptions of point pressure, an important area for further work is to determine how the subjective assessment of pressure in a match correlates with probabilistic importance.

Although we found improved discrimination and predictive performance with clutch statistics for both the men's and women's tours, there were some

interesting gender differences in performance measures with the strongest effects. We found stronger discrimination in several serve statistics (first serve in and aces) for men compared to women, whereas women had greater discrimination in winners. Predictive performance was also stronger for most of the women's matches, though this could be explained by their shorter match format. The physical differences of the men's and women's studies have received considerable attention (Fernandez-Fernandez et al., 2009; Hizan et al., 2011). The clutch statistics create new opportunities to delve more into possible gender differences in how in-match performance is affected by game pressure.

7. CONCLUSIONS

Clutch averaging is a broadly applicable method that accounts for variable point importance when assessing performance in tennis. The method can be applied to any point-level events during a match to highlight when a player is performing a skill well (or poorly) in the most critical situations. Clutch statistics are not only easy to understand they also do better at differentiating the performance characteristics of winners and losers of matches compared to conventional averages. Another major strength of the method is its simplicity of implementation, which will be key to its adoption by the tennis industry. Together, these strengths make clutch averaging a promising tool for advancing performance analysis in tennis.

REFERENCES

- Afremow, J. (2015). *The Champion's Mind: How Great Athletes Think, Train, and Thrive*. Rodale Books, Emmaus, Pennsylvania.
- Fernandez-Fernandez, J., Sanz-Rivas, D. and Mendez-Villanueva, A. (2009). A review of the activity profile and physiological demands of tennis match play. In *Strength & Conditioning Journal*, 31 (4): 15–26.
- Goldstein, J.H. (1979). *Sports, games, and play: social and psychological viewpoints*. Hillsdale, N.J.: L. Erlbaum Associates ; New York : distributed by Halsted Press Division, Wiley. URL <http://0-search.ebscohost.com.library.vu.edu.au/login.aspx?direct=true&db=cat02404a&AN=vic.b1065165&site=eds-live>.
- González-Díaz, J., Gossner, O. and Rogers, B.W. (2012). Performing best when it matters most: Evidence from professional tennis. In *Journal of Economic Behavior & Organization*, 84 (3): 767–781.
- Hibbs, D. (2010). A conceptual analysis of clutch performances in competitive sports. In *Journal of the Philosophy of Sport*, 37 (1): 47–59.
- Hill, D.M. and Shaw, G. (2013). A qualitative examination of choking under pressure in team sport. In *Psychology of Sport and Exercise*, 14 (1): 103–110.

- Hizan, H., Whipp, P. and Reid, M. (2011). Comparison of serve and serve return statistics of high performance male and female tennis players from different age-groups. In *International Journal of Performance Analysis in Sport*, 11 (2): 365–375.
- Keene, J.R. and Cummins, R.G. (2009). Sports commentators and source credibility: Do those who can't play... commentate? In *Journal of Sports Media*, 4 (2): 57–83.
- Klaassen, F.J. and Magnus, J.R. (2001). Are points in tennis independent and identically distributed? evidence from a dynamic binary panel data model. In *Journal of the American Statistical Association*, 96 (454): 500–509.
- Knight, G. and O'Donoghue, P. (2012). The probability of winning break points in grand slam men's singles tennis. In *European Journal of Sport Science*, 12 (6): 462–468.
- Kovalchik, S. and Ingram, M. (2016). Hot heads, cool heads, and tacticians: Measuring the mental game in tennis (ID: 1464). In *10th Annual MIT Sloan Sports Analytics Conference, Boston, MA*.
- Morris, C. (1977). The most important points in tennis. In *Optimal Strategies in Sports*, 5: 131–140.
- Moss, B. and O'Donoghue, P. (2015). Momentum in us open men's singles tennis. In *International Journal of Performance Analysis in Sport*, 15 (3): 884–896.
- O'Donoghue, P.G. (2001). The most important points in grand slam singles tennis. In *Research quarterly for exercise and sport*, 72 (2): 125–131.
- Reid, M., Morgan, S. and Whiteside, D. (2016). Matchplay characteristics of grand slam tennis: implications for training and conditioning. In *Journal of sports sciences*, 34 (19): 1791–1798.
- Solomonov, Y., Avugos, S. and Bar-Eli, M. (2015). Do clutch players win the game? testing the validity of the clutch player's reputation in basketball. In *Psychology of Sport and Exercise*, 16: 130–138.
- Wang, J., Callahan, D. and Goldfine, B. (2003). Choking under pressure in competition and psychological intervention approaches. In *Strength & Conditioning Journal*, 25 (5): 69–75.
- Wei, X., Lucey, P., Morgan, S. and Sridharan, S. (2016). Forecasting the next shot location in tennis using fine-grained spatiotemporal tracking data. In *IEEE Transactions on Knowledge and Data Engineering*, 28 (11): 2988–2997.
- Weinberg, R.S. (2013). *Tennis: Winning the Mental Game*. H.O. Zimman, Lynn, Massachusetts.
- Wright, B., Rodenberg, R.M. and Sackmann, J. (2013). Incentives in best of n contests: Quasi-simpson's paradox in tennis. In *International Journal of Performance Analysis in Sport*, 13 (3): 790–802.