ON THE SIMILARITY OF METHODS FOR RANKING ALTERNATIVES AND MAKING TRANSITIVE THE INDEX NUMBERS

ELVIO MATTIOLI[1], GIUSEPPE RICCIARDO LAMONICA[2]

Department of Economics and Social Science, University "Politecnica delle Marche", Ancona, Italy

Abstract. *This paper presents an empirical analysis of the response given by several methods proposed in literature to rank the alternatives, to determine their weights and to perform accurate comparisons among economic phenomena. The various scaling methods are evaluated using real data provided by the Italian National Institute of Statistics (ISTAT). The most relevant results achieved are the close concordance of the weights obtained with the various methods and the robustness of the evaluation performed.*

Keywords: *Ratio-scale matrices, Pairwise comparison, Preference values, Index numbers, Analytic Hierarchy Process.*

1. INTRODUCTION

The Analytic Hierarchy Process (AHP), proposed by Saaty (1977, 1980), is a commonly applied multi-criteria decision making method (Crowford and Williams, 1985; Carriere and Finster, 1992; Lootsma, 1997; Basak, 2002). Referring to the literature for details, let $A=\{A_1, \ldots, A_n\}$ be the set of n alternatives to be compared, and $\mathbf{C}$ the $n\times n$ matrix whose generic element c_{ij} represent the extent to which alternative A_i is more important than A_j; in this context it is assumed that this extent is expressed on a ratio scale.

At most $n(n-1)/2$ comparisons are required because the following constraint, also known as "reciprocity property", is imposed:

$$c_{ij}=1/c_{ji} \quad i,j = 1,\ldots,n. \tag{1}$$

[1] Elvio Mattioli, email: e.mattioli@univpm.it.

[2] Giuseppe Ricciardo Lamonica, email: g.ricciardo@univpm.it.

The authors would like to thanks two anonymous referees for their detailed and very constructive comments.

For example, if c_{ij}=3 namely A_i is deemed to be three times more important than A_j; therefore the importance of A_j must be one third that of A_i, hence c_{ji}=1/3. It straightforwardly follows from (1) that:

$$c_{ii} = 1 \qquad i = 1,\ldots,n. \tag{2}$$

Matrix C with positive elements possessing property (1) is known as a "positive reciprocal matrix" or "ratio-scale matrix".

It should be pointed out, however, that constraint (1) does not ensure complete (or perfect) consistency, which is obtained if the following relation (transitivity property) is verified:

$$c_{ij}= c_{ik}c_{kj} \qquad \forall\ i, j, k. \tag{3}$$

If the weights w_i (and, therefore, the importance) associated with the alternative A_i (for i=1, …,n) are known, and if matrix **C** is perfectly consistent, one would have:

$$c_{ij}= w_i/w_j \qquad i,j = 1,\ldots,n. \tag{4}$$

Clearly from (4), in case of perfect consistency, the weights w_i would be immediately deducible from any row or column of matrix **C**. In practice, however, rarely this matrix displays a complete consistency.

This issue has prompted the development of various methods to determine the weights by minimising the divergence, suitably defined, between the values c_{ij} and the theoretical ones w_i/w_j or by an estimation process that is known to produce correct results in error-free cases. Many methods have been proposed so far and only some of them have been selected in this study and concisely summarised in Section 2.

A similar problem arises within the methodology aimed at performing accurate comparisons among economic phenomena observed on different occasions, namely the *index number theory*.

Index numbers allow for comparisons among relative differences of a phenomenon, examining their corresponding intensities on different occasions. For example, if we consider the unit price of a generic good on two occasions *A* and *B*, which may be two regions, two periods, and so on, the ratios p_A/p_B and p_B/p_A measure the relative difference in the price on occasion *A* with respect to occasion *B* and vice versa. Simple index numbers obviously satisfy the consistency conditions (3).

However, when the phenomenon is analysed across a number of entities or, to refer to the prices example, when the unit prices of *m* goods are considered, the statistical relationships that simultaneously and succinctly measure the relative diversity of prices in the situations compared are known as "complex index numbers" or simply as "index numbers" (Guarini and Tassinari, 1990; Martini, 1991; Alvaro, 1992; Diewert and Nakamura, 1993; Pedretti, 1999; Vitali, 1999).

Many of the complex indexes proposed and widely used in practice do not permit consistent comparisons. For example, the Laspeyres and Paasche indices do not satisfy even the minimum consistency conditions (1) or, in this contest, the reversibility situations property.

This limitation has important implications for the calculation of purchasing power parities. Moreover, the theoretical and practical importance of comparisons among complex index numbers is certainly not restricted to price index numbers; indeed there is an immediate logical symmetry between the latter and the index numbers of quantities.

Formally, considered n situations to be compared, each one characterised by m goods described by the following vectors of prices and quantities: $\mathbf{p}_i=[p_{i1}, p_{i2}, \ldots, p_{im}]$, $\mathbf{q}_i=[q_{i1}, q_{i2}, \ldots, q_{im}]$ $(i=1,\ldots,n)$ and let $\mathbf{C}$ be the matrix of binary comparisons whose elements are, for example, the popular Fisher price index numbers, that is:

$$c_{ij} = \left(\frac{\sum_{k=1}^{m} p_{ik} q_{jk}}{\sum_{k=1}^{m} p_{jk} q_{jk}} \cdot \frac{\sum_{k=1}^{m} p_{ik} q_{ik}}{\sum_{k=1}^{m} p_{jk} q_{ik}} \right)^{1/2} \qquad i,j = 1,\ldots,n. \tag{5}$$

As previously mentioned, these indices are not each other consistent; therefore, also in this case and similarly in the AHP, several methods have been introduced to obtain the weights that are associated with the n situations to compare so that the c_{ij} elements are transitive.

In Section 3 we briefly review the most common methods for the construction of complex and consistent index numbers. In Section 5, an application of the methods discussed in the Sections 2 and 3 is analysed with the purpose to highlight the close connection or coincidence of the results obtained by the two sets of methods.

In this work we consider data provided by the Italian National Institute of Statistics (ISTAT) and in particular the annual statistics on Italian industrial production and manufacturing from 1994 to 2007. Consequently, our analysis is based on real data, and not on simulations, as it is usually the case in this kind of studies (Blankmeyer, 1987; Golany and Kress, 1993; Dodd *et al.*, 1995). Although on one hand this might be considered a limitation, since only a limited number of observations is examined, on the other it has the advantage that situations of little or no practical importance are eliminated, thereby yielding a clearer and more concrete picture. Section 6 provides some concluding remarks.

2. METHODS FOR OBTAINING WEIGHTS FROM RATIO SCALE MATRICES

2.1 *DOMINANT EIGENVALUE METHOD* - DE (SAATY, 1977, 1980)

If the relations (4) hold, then matrix **C** has one single eigenvalue λ, different from zero, whose value is equal to the order of the matrix; moreover the vector of the weights **w** coincides with the eigenvector associated with the dominant eigenvalue λ:

$$\mathbf{Cw} = \lambda\mathbf{w}. \tag{6}$$

Referring for details to Saaty (2003), even if relation (4) doesn't hold, the dominant eigenvalue can be used to evaluate the vector of weights. Perron-Frobenius theorem ensures the positivity of the components of **w** and therefore its acceptability in the context considered.

Moreover, it is possible to measure the degree of approximation associated with this evaluation by means of the following index known as the 'consistency index' (Saaty, 1977):

$$\mathrm{I}(\mathbf{C}) = (\lambda\text{-}n)/(n\text{-}1). \tag{7}$$

2.2 *MODIFIED DOMINANT EIGENVALUE METHOD* - MDE (COGGER AND YU, 1985)

This method is a modification of the previous one. Bearing in mind that matrix **C** is reciprocal, this technique considers only the upper triangle. Using **T** to denote a matrix, such that:

$$t_{ij} = \begin{cases} c_{ij} & \text{if } i \geq j \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

and **G** to denote the diagonal matrix with the elements:

$$g_{ii} = n\text{-}i\text{+}1 \qquad i = 1,\ldots,n. \tag{9}$$

then the estimates of weights correspond to the solution of the system:

$$(\mathbf{G}^{-1}\,\mathbf{T} - \mathbf{U})\mathbf{w} = \mathbf{0} \qquad (10)$$

where **U** is the unitary matrix. The solution is obtained recursively by means of the relations:

$$\mathrm{w}_i = \sum_{j=i+1}^{n} \mathrm{c}_{ij}\mathrm{w}_j / (n-1) \qquad i = 1,\ldots,n-1\,. \qquad (11)$$

which are easy to compute. It should be noted that Ishizaka and Lusti (2006), considering a previous comparative study (Takeda *et. al*., 1987) in which only one evaluation criteria is used, do not advice the application of this method since, using only the upper triangle of the matrix to calculate the priorities, the ranking depends on the order of the alternatives in the matrix and the method may be not reliable. Since in literature there is no consensus to which criteria are most appropriate for the evaluation of the various techniques for deriving weight (Golany and Kress, 1993) we decided to consider in our analysis this technique also in view of the obtained results.

2.3 *DIRECT LEAST SQUARES METHOD* - DLS (SAATY AND VARGAS, 1984)

It is the classic least squares method according to which the weights are determined in order to minimize the objective function:

$$\varphi_1(\mathbf{w}) = \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\mathrm{c}_{ij} - \frac{\mathrm{w}_i}{\mathrm{w}_j}\right)^2. \qquad (12)$$

In this case the solution **w** does not have an analytical explicit expression but must be calculated with iterative techniques.

2.4 *WEIGHTED LEAST SQUARES METHOD* -WLS (CHU *ET AL.*, 1979)

This method is similar to the previous one but differs from it in the weighting given to square deviations. In this case the objective function to minimize takes the following form:

$$\varphi_2(\mathbf{w}) = \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\mathrm{c}_{ij} - \frac{\mathrm{w}_i}{\mathrm{w}_j}\right)^2 w_j^2 = \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\mathrm{c}_{ij}\mathrm{w}_j - \mathrm{w}_i\right)^2 \qquad (13)$$

Different from the previous DLS method, which may have multiple solutions, (13) admits only one and strictly positive solution. This can be straightforwardly obtained by solving the following system of linear equations:

$$\left(\sum_{i\neq k}^{n} c_{ik}^2 + n - 1\right) w_k - \sum_{j\neq k}^{n}\left(c_{ij} + c_{kj}\right) w_j + \lambda = 0 \qquad k=1, \ldots, n \tag{14}$$

generated by the first-order conditions for the Lagrange auxiliary function constructed by taking (13) as the objective function and constraining the weights to unitary sum.

2.5 *LOGARITHMIC LEAST SQUARES METHOD* -LLS (SAATY AND VARGAS, 1984)

Given the multiplicative nature of the positive reciprocal matrix **C** (consider the fundamental relation (3)), it seems natural to measure the divergences between the observed values c_{ij} and the theoretical values w_i/w_j through their ratio, that is by calculating the relative error measured by the deviation of this ratio from unity.

Considering, for simplicity, the logarithm of these ratios and the least squares method, the objective function becomes:

$$\varphi_3(\mathbf{w}) = \sum_{i=1}^{n}\sum_{j=1}^{n}\left(ln\frac{c_{ij}}{w_i / w_j}\right)^2 = \sum_{i=1}^{n}\sum_{j=1}^{n}\left(ln c_{ij} - ln w_i - ln w_j\right)^2. \tag{15}$$

Selecting the arbitrary multiplying constant of the weights so that their product is unitary, the components of vector **w** that minimise (15) correspond to the geometric mean of the elements in the corresponding row of matrix **C**:

$$w_i = \left(\prod_{j=1}^{n} c_{ij}\right)^{\frac{1}{n}} \qquad i=1, \ldots, n \tag{16}$$

3. METHODS TO CONSTRUCT COMPLEX AND CONSISTENT INDEX NUMBERS

To construct complex and consistent index numbers, we assume that n situations must be compared, each of them characterised by m goods described by the following vectors of prices and quantities: $\mathbf{p}_i=[p_{i1}, p_{i2}, \ldots, p_{im}]$, $\mathbf{q}_i=[q_{i1}, q_{i2}, \ldots, q_{im}]$ $(i=1,\ldots,n)$.

3.1 *THE GINI-ELTETO-KÖVES-SZULC METHOD* - GEKS (GINI, 1924; ELTETÖ AND KÖVES, 1964; SZULC, 1964)

It is based on the matrix **C** of binary comparisons whose elements are the Fisher price index numbers defined in (5). The logarithmic least squares method (LLS) mentioned earlier has been proposed in order to provide consistent comparisons expressed by the **C** matrix, which is evidently positive reciprocal.

3.2 *THE TÖRNQUIST THEIL METHOD* -TT (TÖRNQUIST, 1936; THEIL,1973)

In this case too, considerations are made for matrix **C** of binary comparisons, which are then made consistent by means of the logarithmic least squares method. Unlike the previous case, the elements that constitute the matrix are Törnquist Theil bilateral indices:

$$c_{ij} = \prod_{k=1}^{m} \left(\frac{p_{ik}}{p_{jk}} \right)^{\alpha_{ijk}} \qquad i,j = 1,\dots,n. \tag{17}$$

where:

$$\alpha_{ijk} = \frac{1}{2} \left(\frac{p_{ik} q_{ik}}{\sum_{k=1}^{m} p_{ik} q_{ik}} + \frac{p_{jk} q_{jk}}{\sum_{k=1}^{m} p_{jk} q_{jk}} \right). \tag{18}$$

3.3 *THE ECONOMIC COMMISSION FOR LATIN AMERICA METHOD* - ECLA (GINI, 1924)

This method too can be traced back to the work of Gini (1924), and it has been used by the Economic Commission for Latin America. Its distinctive feature is that it directly and consistently constructs the **C** matrix of comparisons, whose elements are defined in the following manner:

$$c_{ij} = \left(\sum_{k=1}^{m} p_{ik} \bar{q}_k \right) \Big/ \left(\sum_{k=1}^{m} p_{jk} \bar{q}_k \right) \qquad i,j = 1,\dots,n, \tag{19}$$

where:

$$\bar{q}_k = \frac{1}{n} \sum_{i=1}^{n} q_{ik} \tag{20}$$

represents the mean of the quantities of the *k*-th good belonging to the set of statistical units considered. It is evident that any linear combination of the quantities q_{ik}, with positive weights, gives rise to consistent comparisons.

3.4 *THE GEARY-KHAMIS METHOD* - GK (GEARY, 1958; KHAMIS, 1972)

This method does not require the construction of a consistent matrix of binary comparisons. Rather, it is constructed directly, by means of (3), from the weights, w_i's, which, in the context treated by Geary and Khamis, act as conversion factors.

The technique is iterative and is divided into two phases. In the first, the conversion factors (or weights) w_i are used to determine the mean price (π_k) of each good in the basket of the statistical collective:

$$\pi_k = \frac{\sum_{i=1}^{n} w_i p_{ik} q_{ik}}{\sum_{i=1}^{n} q_{ik}} \qquad k = 1,\ldots,m. \tag{21}$$

In the second phase, the conversion factors are determined by:

$$w_i = \frac{\sum_{k=1}^{m} \pi_k q_{ik}}{\sum_{k=1}^{m} p_{ik} q_{ik}} \qquad i = 1,\ldots,n. \tag{22}$$

Each iteration involves two steps: firstly, after assigning to w_i an arbitrary initial value, (21) is used to determine the mean prices; secondly, the latter are inserted in (22) to determine the conversion factors (weights). The procedure is iterated until the solutions of two successive iterations are considered equal.

3.5 *THE GERARDI METHOD* - G (GERARDI, 1978)

This technique does not differ substantially from the GK method. The only difference is that in this case the mean prices of the individual goods are calculated using a geometric mean of the prices observed on the *n* occasion:

$$\pi_k = \left(\prod_{i=1}^{n} p_{ik} \right)^{1/n} \qquad k = 1,\ldots,m. \tag{23}$$

4. ADVANTAGES AND DRAWBACKS OF EACH METHOD REVIEWED

The methods reviewed are derived from different underling concepts and have different strengths and weaknesses. Tab. 1 gives the summary of the estimation methods: (a) whether it is an optimization method; (b) whether it is an iterative method; (c) whether the minimising objective is a distance function; (d) whether an "a priori" matrix **C** is required.

Table 1: Properties of estimation methods.

Properties	Methods									
	DE	**MDE**	**DLS**	**WLS**	**LLS**	**GEKS**	**TT**	**ECLA**	**GK**	**G**
Optimization	No	No	Yes	Yes	Yes	Yes	Yes	No	No	No
Iterative	No	No	Yes	No	No	No	No	No	Yes	No
Distance	No	No	Yes	Yes	Yes	Yes	Yes	No	No	No
a priori C	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	No	No

As regards the construction of complex and consistent index numbers, it seems useful to highlight the following strengths in the case of methods GEKS and TT, and weaknesses in the case of the methods ECLA and G.

- GEKS: the elements of matrix C are index numbers that meet the factor-reversal test (Fisher "ideal index numbers");
- TT: the elements of matrix **C** are index numbers that are particularly relevant in the economic approach for the construction of index numbers;
- ECLA: the drawback of this method is that the consistency was achieved by averaging the quantities q_{ik} (see formula (20)). This is only one of the possible choices. Indeed, any linear combination of the q_{ik} with positive weights gives consistent indices;
- G: the index doesn't take into account the quantities exchanged.

5. EMPIRICAL ANALYSIS

In order to examine the behaviour of the methods previously presented and then to highlight the common characteristics and those specific we carried out a series of experiments on empirical data from official sources.

The data used are the annual statistics on Italian industrial production and manufacturing from 1994 to 2007 published by ISTAT (1999-2010).

Following the Nace Rev. 1 classification, the Italian industrial production and manufacturing is classified in 22 economics sectors (or division). Each one of them is further divided in groups, afterwards in classes and finally in economic activity categories. In total, every year, at least 4000 categories are recorded.

For each economic activity category the price and quantity sold are available.

It should also be pointed out that the data series are not perfectly comparable, and that, for our purposes, we decided to eliminate all categories that: were missing in all years, were not published in some years because covered by statistical confidentiality, were not distinguished according to prices and quantities sold, and had been surveyed over the years using different measurement systems and therefore could not be related to the same unit of measurement.

This screening procedure led to the exclusion of the sector relating to the *production and sale of office machinery, computers and information systems*, and the sector of the *production and sale of tobacco products*.

Moreover, as a synthesis measure of the results obtained, we considered the following consistency index, that is, the mean squared deviation (MSD) of the weights with respect to those calculated as a mean of the nine methods applied:

$$\mathrm{MSD}_h = \left[\frac{1}{n}\sum_{i=1}^{n}\left(\mathrm{w}_{ih} - \bar{\mathrm{w}}_i\right)^2\right]^{1/2} \qquad h = 1,\ldots,9, \tag{24}$$

where:

$$\bar{\mathrm{w}}_i = \frac{1}{9}\sum_{h=1}^{9}\mathrm{w}_{ih} \qquad i = 1,\ldots,n. \tag{25}$$

Considering the first economic sector (Production and sale of non-energy-producing minerals) and starting from its economic activity category, two temporal binary comparison matrix of the sector production was constructed: the first using Fisher price index numbers, and the second through the Törnqvist-Theil price index numbers.

Since these two matrices are extremely similar, we report (Tab. 2), by way of example, only the Fisher index numbers for the economic sector in consideration.

The elements of this matrix show the temporal evolution of the sector price. In particular, the generic element ($F_{t;s}$) is the relative variation of the price between times s and t. For example, $F_{2002;1994}$= 0.87 show that between the 1994 and 2002 the sector prices decreased by 13%.

As previously mentioned, these indexes are not consistent, for example $F_{2002;1994} \neq F_{2002;2000} \cdot F_{2000;1994}$ (i.e. $0.87 \neq 0.92 \cdot 0.97$).

Consequently, the DE, MDE, DLS, WLS, LLS and GEKS methods previously considered were used to improve the consistency over the years of the matrices of Fisher index numbers, and the TT method was used to make consistent the Törnqvist-Theil price index numbers.

Finally the ECLA, GK and G methods were directly used to construct a matrix of consistent index numbers.

Table 2: Fisher price index numbers of Sector 1.

Years	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
1994	1.00	1.12	1.06	1.00	0.97	0.96	1.03	1.11	1.15	1.17	1.16	1.24	1.16	1.22
1995	0.89	1.00	0.95	0.92	0.89	0.89	0.96	1.02	1.03	1.06	1.06	1.11	1.05	1.09
1996	0.94	1.06	1.00	0.97	0.91	0.93	1.00	1.07	1.09	1.09	1.09	1.14	1.08	1.13
1997	1.00	1.08	1.03	1.00	0.97	0.97	1.03	1.08	1.13	1.14	1.13	1.17	1.12	1.17
1998	1.03	1.12	1.10	1.03	1.00	1.00	1.07	1.14	1.18	1.18	1.17	1.22	1.16	1.22
1999	1.04	1.12	1.08	1.03	1.00	1.00	1.06	1.12	1.16	1.17	1.16	1.21	1.17	1.22
2000	0.97	1.04	1.00	0.97	0.94	0.94	1.00	1.06	1.09	1.10	1.09	1.14	1.12	1.16
2001	0.90	0.98	0.94	0.92	0.88	0.89	0.94	1.00	1.03	1.03	1.02	1.08	1.04	1.08
2002	0.87	0.97	0.92	0.89	0.85	0.86	0.92	0.97	1.00	1.00	0.99	1.05	1.01	1.06
2003	0.85	0.94	0.91	0.88	0.85	0.86	0.91	0.97	1.00	1.00	0.99	1.05	1.01	1.05
2004	0.86	0.94	0.91	0.88	0.86	0.86	0.92	0.98	1.01	1.01	1.00	1.06	1.03	1.06
2005	0.81	0.90	0.88	0.86	0.82	0.83	0.87	0.93	0.96	0.96	0.94	1.00	0.98	1.01
2006	0.86	0.96	0.93	0.90	0.86	0.85	0.90	0.96	0.99	0.99	0.97	1.02	1.00	1.03
2007	0.82	0.91	0.89	0.86	0.82	0.82	0.86	0.93	0.95	0.95	0.94	0.99	0.98	1.00

In Tab. 3, normalised to the baseline year, are shown the weights obtained by the nine methods considered, while in Tab. 4 are shown the new consistent index numbers obtained considering, by way of example, only the weights of the DE methods.

Table 3: Weights determined by the different methods to make consistent price index numbers of Sector 1.

Years	DE	MDE	DLS	WLS	LLS GEKS	GK	G	TT	ECLA
1994	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1995	0.907	0.908	0.907	0.907	0.907	0.939	0.935	0.911	0.880
1996	0.945	0.944	0.945	0.946	0.945	0.947	0.949	0.930	0.939
1997	0.979	0.977	0.979	0.980	0.979	0.952	0.964	0.958	0.996
1998	1.018	1.017	1.018	1.019	1.018	0.985	0.996	0.999	1.046
1999	1.013	1.009	1.013	1.014	1.013	0.975	0.986	0.996	1.046
2000	0.953	0.953	0.953	0.954	0.953	0.923	0.930	0.940	0.984
2001	0.896	0.895	0.896	0.897	0.896	0.864	0.870	0.880	0.930
2002	0.870	0.869	0.870	0.871	0.870	0.845	0.850	0.857	0.895
2003	0.865	0.869	0.865	0.865	0.865	0.844	0.850	0.851	0.891
2004	0.872	0.879	0.872	0.872	0.872	0.849	0.851	0.857	0.900
2005	0.829	0.832	0.829	0.830	0.829	0.803	0.804	0.816	0.862
2006	0.861	0.851	0.861	0.862	0.861	0.833	0.839	0.844	0.884
2007	0.829	0.829	0.828	0.829	0.829	0.802	0.802	0.816	0.858
MDS	0.004	0.005	0.004	0.005	0.004	0.023	0.018	0.011	0.030

Table 4: Consistent new indices number obtained with the DE method.

Years	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
1994	1.00	1.10	1.06	1.02	0.98	0.99	1.05	1.12	1.15	1.16	1.15	1.21	1.16	1.21
1995	0.91	1.00	0.96	0.93	0.89	0.90	0.95	1.01	1.04	1.05	1.04	1.09	1.05	1.09
1996	0.95	1.04	1.00	0.97	0.93	0.93	0.99	1.05	1.09	1.09	1.08	1.14	1.10	1.14
1997	0.98	1.08	1.04	1.00	0.96	0.97	1.03	1.09	1.13	1.13	1.12	1.18	1.14	1.18
1998	1.02	1.12	1.08	1.04	1.00	1.00	1.07	1.14	1.17	1.18	1.17	1.23	1.18	1.23
1999	1.01	1.12	1.07	1.03	1.00	1.00	1.06	1.13	1.16	1.17	1.16	1.22	1.18	1.22
2000	0.95	1.05	1.01	0.97	0.94	0.94	1.00	1.06	1.10	1.10	1.09	1.15	1.11	1.15
2001	0.90	0.99	0.95	0.92	0.88	0.88	0.94	1.00	1.03	1.04	1.03	1.08	1.04	1.08
2002	0.87	0.96	0.92	0.89	0.85	0.86	0.91	0.97	1.00	1.01	1.00	1.05	1.01	1.05
2003	0.87	0.95	0.92	0.88	0.85	0.85	0.91	0.97	0.99	1.00	0.99	1.04	1.00	1.04
2004	0.87	0.96	0.92	0.89	0.86	0.86	0.92	0.97	1.00	1.01	1.00	1.05	1.01	1.05
2005	0.83	0.91	0.88	0.85	0.81	0.82	0.87	0.93	0.95	0.96	0.95	1.00	0.96	1.00
2006	0.86	0.95	0.91	0.88	0.85	0.85	0.90	0.96	0.99	1.00	0.99	1.04	1.00	1.04
2007	0.83	0.91	0.88	0.85	0.81	0.82	0.87	0.93	0.95	0.96	0.95	1.00	0.96	1.00

The most striking aspect that emerges from Tab. 3 is the close concordance and in some cases the perfect coincidence of the results obtained when applying the various methods.

This analysis was then repeated for the remaining 19 economic sectors and the results, synthesised through the MSD consistency index (the detailed results are available from the authors upon request) are set out in Tab. 5. Leaving further comparisons to the reader, here we merely point out that the most important result yielded by this large series of analysis, as well as highlighted by the MDS index which in the majority of cases has extremely low values, is the *close conformity of the weights values obtained with the various methods of calculations*. Only the ECLA, GK and G methods display weights values in some sectors that are slightly different from other methods, whilst in other situations, such as sectors 6, 9 and 12, these differences are more substantial. These methods are those that do not presuppose a matrix of binary comparisons to be adjusted.

Summing up, the analysis highlights the high degree of concordance among the weights' values obtained with the two sets of methods.

Table 5: MDS consistency index by economic sectors and methods.

Economic Sectors [*]	**DE**	**MDE**	**DLS**	**WLS**	**LLS GEKS**	**GK**	**G**	**TT**	**ECLA**
Sector 1	0.004	0.005	0.004	0.005	0.004	0.023	0.018	0.011	0.030
Sector 2	0.001	0.002	0.002	0.002	0.001	0.013	0.009	0.003	0.016
Sector 4	0.005	0.007	0.006	0.006	0.005	0.064	0.049	0.006	0.078
Sector 5	0.007	0.009	0.013	0.014	0.008	0.065	0.043	0.007	0.068
Sector 6	0.006	0.007	0.009	0.006	0.007	0.051	0.022	0.010	0.049
Sector 7	0.057	0.054	0.158	0.064	0.057	0.145	0.140	0.122	0.119
Sector 8	0.001	0.002	0.001	0.001	0.001	0.010	0.010	0.003	0.013
Sector 9	0.003	0.008	0.001	0.003	0.002	0.017	0.020	0.003	0.028
Sector 10	0.060	0.063	0.348	0.068	0.059	0.165	0.171	0.049	0.061
Sector 11	0.016	0.022	0.013	0.015	0.015	0.061	0.061	0.058	0.066
Sector 12	0.028	0.028	0.028	0.028	0.028	0.016	0.091	0.095	0.149
Sector 13	0.004	0.004	0.003	0.004	0.004	0.025	0.020	0.003	0.029
Sector 14	0.014	0.011	0.013	0.014	0.014	0.044	0.098	0.063	0.064
Sector 15	0.005	0.006	0.005	0.005	0.005	0.044	0.035	0.006	0.057
Sector 17	0.059	0.060	0.054	0.052	0.058	0.353	0.227	0.138	0.455
Sector 18	0.051	0.046	0.055	0.054	0.051	0.322	0.096	0.128	0.282
Sector 19	0.033	0.023	0.031	0.033	0.032	0.113	0.051	0.060	0.080
Sector 20	0.034	0.047	0.029	0.039	0.035	0.177	0.246	0.069	0.197
Sector 21	0.002	0.005	0.002	0.002	0.002	0.011	0.012	0.003	0.017
Sector 22	0.007	0.015	0.008	0.006	0.008	0.104	0.071	0.016	0.133

[*] See Appendix

6. CONCLUSION

In this paper we have emphasised the close connection between two approaches: the first one that uses subjective preference judgements, and the second one based on objective data regarding prices and quantities. Using empirical data, which are more incontrovertible, we have conducted a series of experiments using time series data.

The most significant results are the close concordance of the weights obtained with the various methods and the notable robustness of the evaluations performed.

An immediate conclusion that can be drawn from our analysis is that these methods should be more widely used. Which method in particular should be selected is of little importance from the practical point of view, given the extremely close concordance obtained in the analyses. More specifically, focusing on index numbers, we suggest that those (Laspeyres, Paasche, Fisher, Theil) that are incoherent in multiple comparisons should be discarded. Those index numbers were widely used in the past because of their computational simplicity, which nowadays is an irrelevant problem.

APPENDIX

Economic Sectors	Descriptions
Sector 1	Production and sale of non-energy-producing minerals;
Sector 2	Production and sale of food and beverages;
Sector 4	Production and sale of textiles;
Sector 5	Production and sale of garments and furs;
Sector 6	Production and sale of leather and leather goods;
Sector 7	Production and sale of wood and wooden products, cork, etc.;
Sector 8	Production and sale of paper-making pulp. paper and paper products;
Sector 9	Production and sale of printed and recorded products;
Sector 10	Production and sale of chemicals and man-made fibres;
Sector 11	Production and sale of rubber and plastic articles;
Sector 12	Production and sale of non-metallic mineral products;
Sector 13	Production and sale of metals and alloys;
Sector 14	Production and sale of machinery and equipment;
Sector 15	Production and sale of office machinery, computers and information systems;
Sector 17	Production and sale of electrical machinery and appliances;
Sector 18	Production and sale of radio and television apparatus;
Sector 19	Production and sale of surgical and orthopaedic products;
Sector 20	Production and sale of vehicles, trailers and engines;
Sector 21	Production and sale of other means of transport;
Sector 22	Production and sale of furniture and other manufactures.

REFERENCES

Alvaro, G. (1992). *Contabilità nazionale e statistica economica*. Cacucci Editore, Bari.

Basak, I. (2002). On the use of information criteria in analytic hierarchy process. *European Journal of Operational Research*, (141): 200-216.

Blankmeyer, E. (1987). Approaches to consistency adjustments. *Journal of Optimization Theory and Applications,* (54): 479-488.

Carriere, J. and Finster, M. (1992). Statistical theory for ratio model of paired comparisons. *Journal of Mathematical Psychology,* (36): 450-460.

Cogger, K.O. and Yu, P.L. (1985). Eigenweight vectors and least distance approximation for revealed preference in pairwise weight ratios. *Journal of Optimization Theory and Applications,* (46): 483-491.

Chu, A.T., Kalaba, R.E. and Spingarn, K. (1979). A comparison of two methods for determining the weights of belonging to fuzzy sets. *Journal of Optimization Theory and Applications,* (27): 531-538.

Crowford, G. and Williams, C. (1985). A note on the analysis of subjectives judgements matrices. *Journal of the Mathematical Psychology,* (29): 387-405.

Diewert, W.E. and Nakamura, A.O. (1993). *Essays in index number theory vol. 1.* North Holland, Amsterdam.

Dodd, F.J., Donegan, H.A. and McMaster, T.B.M. (1995). Reassessment of consistency criteria in judgment matrices. *The Statistician,* (44): 31-41.

Eltetö, O. and Köves, T. (1964). One index computation problem of international comparison. *Statisztikai Szemle,* (42): 507-518.

Geary, R.G. (1958). A note on the comparison of exchange rate and purchasing power between countries. *Journal of the Royal Statistical Society series A,* (121): 97-99.

Gerardi, D. (1978). Alcuni aspetti metodologici riguardanti il calcolo delle parità di potere di acquisto tra i paesi della comunità europea. *Atti della XXIX Riunione Scientifica della Società Italiana di Statistica,* vol.2: 171-178. CLUEB, Bologna.

Gini, C. (1924). Quelques considération ou sujet de la construction des nombres indices des prix et des question analogues. *Metron*, (4): 3-162.

Golany, B. and Kress, M. (1993). A multicriteria evaluation of methods for obtaining weights from ratio-scale matrices. *European Journal of Operational Research,* (69): 210-220.

Guarini, R. and Tassinari, F. (1990). *Statistica economica. Problemi e metodi di analisi*. Il Mulino, Bologna.

Ishizaka, A. and Lusti, M. (2006). How to derive priorities in AHP: A comparative study. *Central European Journal of Operations Research*, (14): 387-400.

Istat (1999). *Statistica annuale della produzione industriale. Anni 1994 1995.* Serie informazioni n. 5. Istat, Roma.

Istat (2000). *Statistica annuale della produzione industriale. Anno1996*. Informazioni n. 45. Istat, Roma.

Istat. (2001). *Statistica annuale della produzione industriale. Anno 1997*. Informazioni n. 10. Istat, Roma.

Istat. (2001). *Statistica annuale della produzione industriale. Anno 1998*. Informazioni n. 34. Istat, Roma.

Istat (2002). *Statistica annuale della produzione industriale. Anno 1999.* Informazioni n. 16. Istat, Roma.

Istat (2003). *Statistica annuale della produzione industriale. Anno 2000*. Informazioni n. 1. Istat, Roma.

Istat (2004). *Statistica annuale della produzione industriale. Anno 2001*. Informazioni n. 4. Istat, Roma.

Istat. (2004). *Statistica annuale della produzione industriale. Anno 2002*. Informazioni n. 37. Istat, Roma.

Istat (2006). *Statistica annuale della produzione industriale. Anno 2003*. Informazioni n. 5. Istat, Roma.

Istat (2006). *Statistica annuale della produzione industriale. Anno 2004*. Data available at website: www.istat.it/archivio/13543.

Istat (2007). *Statistica annuale della produzione industriale. Anno 2005*. Data available at website: www.istat.it/archivio/13524.

Istat (2008). *Statistica annuale della produzione industriale. Anno 2006*. Data available at website: www.istat.it/archivio/13432.

Istat (2010). *Statistica annuale della produzione industriale. Anno 2007*. Data available at website: www.istat.it/archivio/9815.

Khamis, S.H. (1969). A new system of index number for national and international purposes. *Journal of the Royal Statistical Society series A*, (135): 96-12.

Lootsma, F.A. (1997). Multicriteria decision analysis in a decision tree. *European Journal of Operational Research*, (101): 442-451.

Martini, M. **(**1991). *Il confronto dei valori nello spazio: un approccio assiomatico*. In Marbach G. (Ed.), *Statistica economica*. UTET, Torino: 301-334.

Pedretti, A. (1999). *I numeri indici. Teoria e pratica*. Giuffrè Editore, Milano.

Saaty, T.L. (1977). A scaling method for priorities in hierarchical structures. *Journal of Mathematical Psychology*, (15): 234-281.

Saaty, T.L. (1980). *The Analytic Hierarchy Process*. McGraw Hill, New York.

Saaty, T.L. (2003). Decision-making with the AHP: Why is the principal eigenvector necessary. *European Journal of Operational Research*, (145): 85-91.

Saaty, T.L. and Vargas, L.G. (1984). Comparison of eigenvalues, logarithmic least squares and squared methods in estimating ratios. *Mathematical Modelling*, (5): 309-324.

Szulc, B. (1964). Index number for multilateral regional comparison. *Przeglad Statystieny*, (3): 239-254.

Takeda, E., Cooger, K.O. and Yu, P.L. (1987). Estimating criterion weights using eigenvectors: A comparative study. *European Journal of Operational Research*, (29): 360-369.

Theil, H. (1973). A new index number formula. *The Review of Economics and Statistics*, (55): 498-502.

Törnqvist, L. (1936). The Bank of Finland’s consumption price index. *Bank of Finland Monthly Bulletin* (10): 1-8.

Vitali, O. (1999). *Statistica economica*. Cacucci Editore, Bari.