

## **ORGANIZATIONAL PERFORMANCE MEASURING: AN APPLICATION OF PANEL DATA MODELS FROM FACTOR ANALYSIS**

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**Abstract.** *Performance measurement is a result of the need organizations have to ensure the appropriate use of resources in the context of strategic planning. Performance measurement should be carried out both transversely and chronologically; the latter measures the evolution of temporal dynamics that, in the presence of databases of great size and complexity, and with correlation and dependence structures, require the use of dimension reduction multivariate methods such as Factor Analysis and Principal Components Analysis, which require fixing the variance-covariance matrix in order to obtain scores, in the case of Factor Analysis, and components, in the case of Principal Components Analysis. In Colombia, the performance for the State University System has been measured since 2003, obtaining a series of capability and results indicators used to measure the 32 public universities of the country. This is the case study in this research, in which, the use of various types of variance-covariance matrices in dimension reduction methods for panel data models, which measure the aforementioned temporal dynamics, are assessed and illustrated.*

**Keywords:** *Benchmarking, Multicollinearity, Panel Data Models, Performance Measurement in Universities, Variance and Covariance Matrices.*

### **1. INTRODUCTION**

New competitive realities and strategies have created an increasing need for organizations to measure their performance as a tool to support decision-making processes and improve financial performance. Performance measurement may

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have several objectives: directly ensuring the appropriate use of resources, evaluating and directing towards the achievement of strategic objectives, and/or assessing managerial performance (Ittner and Larcker, 2003). Neely (1999) suggests seven main reasons for measuring performance:

1. Changes in the nature of work
2. Increased competitiveness
3. Continuous improvement
4. National and international quality awards
5. Changes in organizational roles
6. Changes in the external demand
7. The power of information technology

Many organizations make of performance measurement an essential part of planning and control, and also have information systems used to build databases, which in many cases allow analyzing and controlling processes in real time, so that the analysis of these data and the proper use of the resulting information, support reliable decision-making processes. Nevertheless, in organizations are often heard comments such as “we measure everything what walks and moves, but nothing that matters” (Neely, 1999). This may happen when the measurement is not aligned to the strategy, objectives and performance measures are not clearly and precisely set out, relationships among measures have not been validated, there are errors, or results have not been analyzed statistically. However, if measurement is only made for internal comparison purposes, it has the disadvantage of creating complacency due to a false sense of security and causing a stronger rivalry which can take up more resources than those invested in market competition (Eccles, 1991). This is the reason why benchmarking is necessary; benchmarking refers to identifying competitors and/or companies in the industry that exemplify the best practices in the activity, function, or process carried out by the organization (Eccles, 1991). But benchmarking must be done in conjunction with statistical analysis of data, in which multivariate statistical methods play a key role in the interpretation of the reality of any phenomenon under study, yielding valid conclusions allowing successful planning, collectively and individually assessment, and control of organizations. This analysis requires building a database containing time series observations for a number of individuals or experimental units, k-variables with correlation and dependence structures, and using panel data models that allow controlling the heterogeneity of the organizations and the dynamic effects on the relationship among performance measures (Elsayed and Paton, 2005).

In some cases, it could happen that the explanatory variables have correlation structures (multicollinearity), and that explained variables present high dimensionality, which creates problems estimating panel data models and, consequently, it makes difficult to know the evolution of the studied systems in time. One solution for this situation is using dimension reduction methods such as Principal Components Analysis (PCA) or Factor Analysis (FA). For this, it is required to use the same covariance matrix, otherwise, the comparison over time is invalid. Another solution is reported by Bai (2013), in which he incorporates the mathematical structure of Factor Analysis in the panel data model, but we did not explore this solution. Other authors such as Anderson (1963) that use factor analysis in time series changing the variance covariance matrix for each period of time, which invalidates the comparison.

The use of the variance-covariance matrix of the first year of observation, denoted by  $S_{base}$  – which has been employed by a group of Statisticians in the context of measuring the performance of the state university system in Colombia – was used and a comparison through simulation, using Monte Carlo experiments, of the pooled matrix, denoted by  $S_{pooled}$ , and recommended by the statistical theory for cases like this, with the average matrix of the variables from the various observation periods, denoted by  $S_{mean}$ , was carried out. Additionally, dimension reduction without fixing the variance-covariance matrix, that is leaving the matrix of each period, denoted by  $S_t$ , both of which are common practices, was performed. Finally, these results were illustrated through a case study, in which a strong performance of the base matrix against other matrices was observed for studies such as the one above described.

## 2. METHODS

### 2.1 THE PANEL DATA MODEL BASED ON FACTOR ANALYSIS

Data sets involving data over time for the same transversal units are very common. In measuring efficiency, capability data are obtained for variables in  $N$  observation units, observed in various  $t$  time periods. These data sets provide valuable information and are very useful because they allow observing the evolution in time for observation units not only individually but also collectively (temporal dynamics), and to characterize the unobserved heterogeneity. Modeling was done using panel data models that interpret and reproduce phenomena such as described above, represented by the functional form in (1).

$$\begin{aligned}
 y_{it} &= \alpha_i + \beta x_{it} + u_{it} & (1) \\
 i &= 1, 2, \dots, N \\
 t &= 1, 2, \dots, T,
 \end{aligned}$$

where  $y_{it}$  represents the explained variable,  $\alpha_i$  the non-observable heterogeneity (non-observable individual effects), considered as a random variable,  $\beta$  the coefficients of the explanatory variables,  $x_{it}$  the explanatory variables, and  $u_{it}$  the random error (Wooldridge, 2010). This same author notes that the key question is whether  $\alpha_i$  is correlated with the explanatory variables. The random effect places the individual effect in the error term, assuming that  $\alpha_i$  is orthogonal to  $x_{it}$ ; that is, the variables are independent and uncorrelated. Fixed effects allow ' to be correlated with  $x_{it}$ . Thus:

$$\begin{aligned}
 \text{cov}(x_{it}, \alpha_i) &= 0 \quad t = 1, 2, \dots, T & \text{Random effects} & (2) \\
 \text{cov}(x_{it}, \alpha_i) &\neq 0 \quad t = 1, 2, \dots, T & \text{Fixed effects} & (3)
 \end{aligned}$$

In the analysis of panel data, it is common to find high dimension databases,  $k = p + q$  variables ( $q =$  explained;  $p =$  explanatory), that exhibit correlation and dependency structures within measurable  $p$ - and  $q$ -variables and between both groups of variables, for  $N$  experimental units, and naturally for  $T$  periods of time. The shortage of degrees of freedom and severe multicollinearity in time-series data often frustrate economists who wish to determine the individual influence of each explanatory variable (Hsiao, 2014).

In these cases, dimension reduction methods such as *Factor Analysis*, first proposed by Spearman (1904), and *Principal Components Analysis*, initially proposed by Pearson (1900), are very useful. Seber (2009) states that dimension reduction consists in transforming  $\mathbf{x}$  in an  $m$ -dimensional vector of smaller dimension  $\mathbf{y}_{(m)}$ , in such a way that for a suitable choice of matrix  $\mathbf{A}_{p \times m}$ ,  $\mathbf{A}\mathbf{y}_{(k)}$ , approximates to  $\mathbf{x}$ .

FA identifies an underlying structure or pattern in a set of multivariate data. These new variables or factors should provide a better and easier understanding and analysis of data, allowing assessing individuals or experimental units (Johnson and Wichern, 2007). The main objective of this method is to explain a set of observed  $p$ -variables by a smaller set, that is to say  $m$ -variables, also called latent or unobserved variables.

Let's suppose a vector  $\mathbf{X}_{p \times 1}$  with mean a vector  $\mu$  and variance-covariance matrix  $\Sigma_{p \times p}$ , the factorial model, as shown in (4), expresses each variable as a linear

combination of underlying common factors  $f_1, f_2, f_m$ , accompanied by an error term that accounts for the portion of each variable that is unique (not shared with other variables) (Rencher, 2003):

$$(\mathbf{X} - \boldsymbol{\mu})_{px1} = \mathbf{L}_{pxm} \mathbf{F}_{mx1} + \boldsymbol{\xi}_{px1}, \quad (4)$$

where  $l_{ij}$  are the factor loadings or weights, and the error term  $\xi_i$  accounts for the specific variability of each observation unit.

The FA model can be estimated using the Principal Factor (PF) or the Maximum Likelihood (ML) methods, among others. ML estimate assumes that the observations  $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n$ , are a random sample of  $N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ , where the following function is maximized to estimate the model:

$$L(\boldsymbol{\mu}, \boldsymbol{\Sigma}) = 2\pi^{-np/2} |\boldsymbol{\Sigma}|^{n/2} e^{-\frac{1}{2} \text{tr}[\boldsymbol{\Sigma}^{-1} (\sum_j^n (x_j - \bar{x})(x_j - \bar{x})^T + (x_j - \boldsymbol{\mu})(x_j - \boldsymbol{\mu})^T)]}. \quad (5)$$

In order to properly interpret the evolution of the panel, N units of observation in T periods of time with  $k = p + q$  measured variables, and a fixed variance and covariance matrix (a single matrix) are required to carry out dimension reduction, and to make the comparison of data models valid. Matrices are shown below:

$$\mathbf{S}_{pooled} = \frac{(n_1 - 1)\mathbf{S}_1 + (n_2 - 1)\mathbf{S}_2 + \dots + (n_k - 1)\mathbf{S}_k}{\sum_{i=1}^k n_i - k}, \quad (6)$$

where  $\mathbf{S}_i$  is the covariance matrix of the i-th sample

$$\mathbf{S}_{base} = \mathbf{S}_{t=1} \quad (7)$$

The matrix of the base period  $\mathbf{S}_{base}$  is based on the theory of index numbers, which is an answer to the need to quantitatively express the behavior of a set of individual variables for which there are no common physical units (Delfino, 2002). Index numbers allow for comparisons over time and/or in space for the purpose of studying variations of a complex phenomenon pertaining to any field of human activity that can be observed and measured.

The matrix  $\mathbf{S}_{mean}$  is a common practice used when, due to the complexity of the phenomenon under study, a fixed matrix in dimension reduction is required in order to analyze the evolution of a system over time using panel data models. The average of the explained p variables in the T periods of time is used to calculate the

covariance matrix of these average values. The same procedure is applied to the variables  $q$  explanatory.

When matrices  $S_t$  are used, one for each period of time, the matrix is not fixed. Thus, having a variance-covariance matrix for each period of time, most likely due to a lack of statistical theory which leads to an improper comparison of the different panels.

The methodology for panel data models from dimension reduction methods begins with a broad set of explained and explanatory variables that generally have correlation and dependency relationships among them, generating high dimensionality in explained variables and multicollinearity in explanatory variables. Therefore, dimension reduction through PCA and FA is performed in both sets of variables separately, using the four matrices previously mentioned. It should be noted that correlation matrices can be used instead of variances covariances matrices. Dimension reduction generates a few non-observable variables, called latent variables, and known as *scores* in FA. An example of this is obtaining only two latent variables, one for the explanatory variables and one for the explained variables. These two vectors are used in the fixed effects panel data model; coefficients and  $p$ -values of the latent variable in the explanatory variables are obtained for each experimental unit; and a ranking is performed.

### 3. RESULTS AND DISCUSSION

The methodology presented below consists of a set of simulations to select the best matrices and subsequently use them in the case study.

#### 3.1 SIMULATION

Comparison of various variance and covariance matrices involves the use of multivariate statistical methods for panel data dimension reduction. These methods are initially applied to a set of simulated panels through the Monte Carlo method (Metropolis and Ulam, 1949), and subsequently to the case study. This simulation produces eight variables,  $p=4$  explained variables and  $q=4$  explanatory variables. Probability distribution of the  $k = p + q$  variables is a multivariate normal distribution ( $\mu=0, \Sigma$ ) with  $N=30$  experimental units and  $T=100$  periods of time. In the cross section, correlations between 0.7 and 0.9 were randomly generated for the variables, and an autoregressive model of order 1 (AR1) was used for autocorrelation. Finally, fixed effects were assumed.

### 3.1.1 TESTS OF EQUALITY OF COVARIANCE MATRICES

In order to analyze the minimum variance properties in the estimators of the different variance-covariance matrices under the study described above, the volumes of the concentration ellipsoids throughout the corresponding generalized variances were calculated, allowing identifying the *best* matrices (Serfling, 2009).

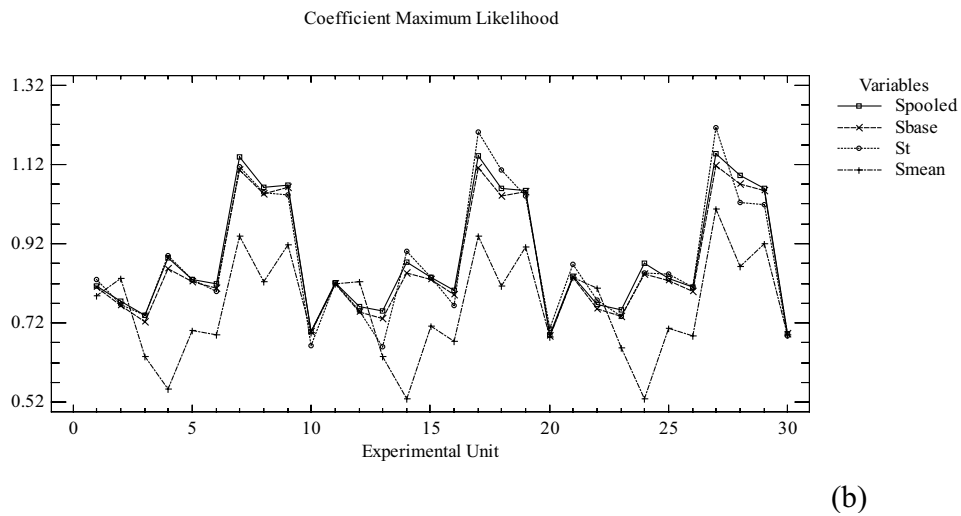
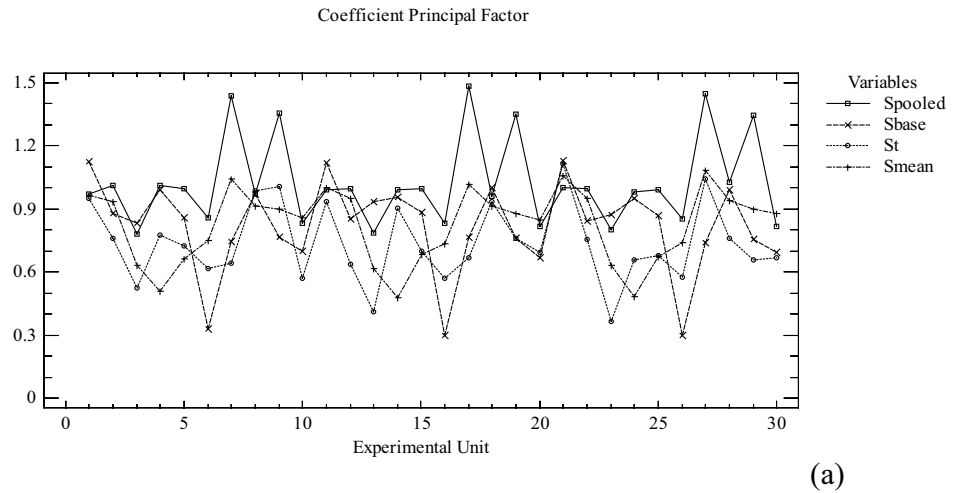
For this, the determinants of the matrices  $\mathbf{S}_{pooled}$ ,  $\mathbf{S}_{base}$ , and  $\mathbf{S}_{mean}$  ( $\mathbf{S}_t$  is not included because it is not a single matrix) were obtained, and the ratio between the determinants of the matrices was calculated using  $\mathbf{S}_{pooled}$  as pattern matrix. Table 1 shows the results. If two matrices are equal, the ratio of their generalized variances is 1; however, as shown in Table 2, all the matrices are different. Moreover, the lowest determinant corresponds to  $\mathbf{S}_{base}$ . As  $|\mathbf{S}_{base}| \leq |\mathbf{S}_{pooled}|$  and  $|\mathbf{S}_{base}| \leq |\mathbf{S}_{mean}|$ , it could be stated that  $\mathbf{S}_{base}$  has a lower concentration ellipsoid and, therefore, is more efficient and is a better estimator of  $\Sigma$ .

**Table 1: Ratio of determinants for simulation**

Matrix	Explanatory Variables Matrix			Explained Variables Matrix		
	$\mathbf{S}_{pooled}$	$\mathbf{S}_{base}$	$\mathbf{S}_{mean}$	$\mathbf{S}_{pooled}$	$\mathbf{S}_{base}$	$\mathbf{S}_{mean}$
Determinant	0.03	0.01	0.01	0.03	0.01	0.01
Ratio	1.00	0.28	0.31	1.00	0.20	0.26

Then, dimension reduction through Factorial Analysis was applied using PF and ML with Varimax rotation (Kaiser, 1958) in order to maximize the variance explained by the factors and, above all, aiming to establish a conceptual identity of common factors or latent variables in the four matrices. Then, scores were used in the fixed effects panel data model. Finally, the estimated coefficient  $\hat{\beta}$ , its standard deviation  $\hat{\sigma}_{\hat{\beta}}$ , and the test statistic  $t$ , were obtained. Figures 1 to 4 show these results.

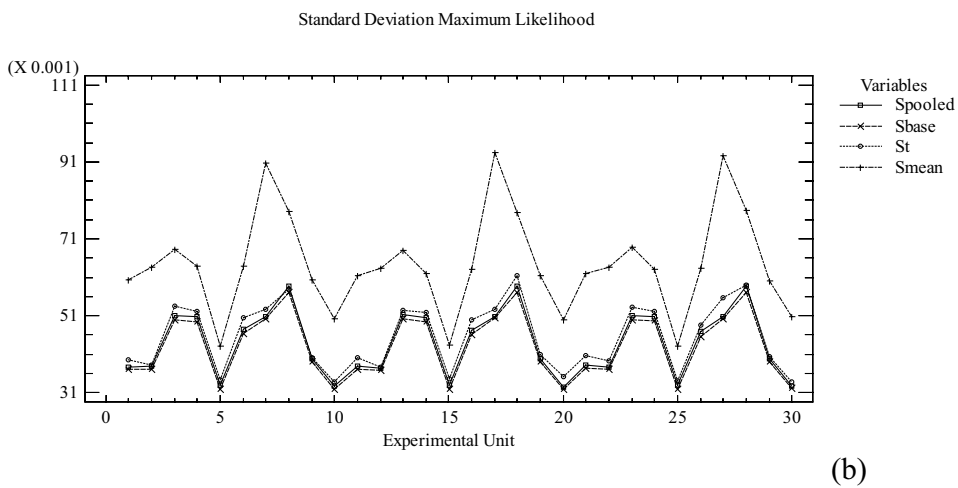
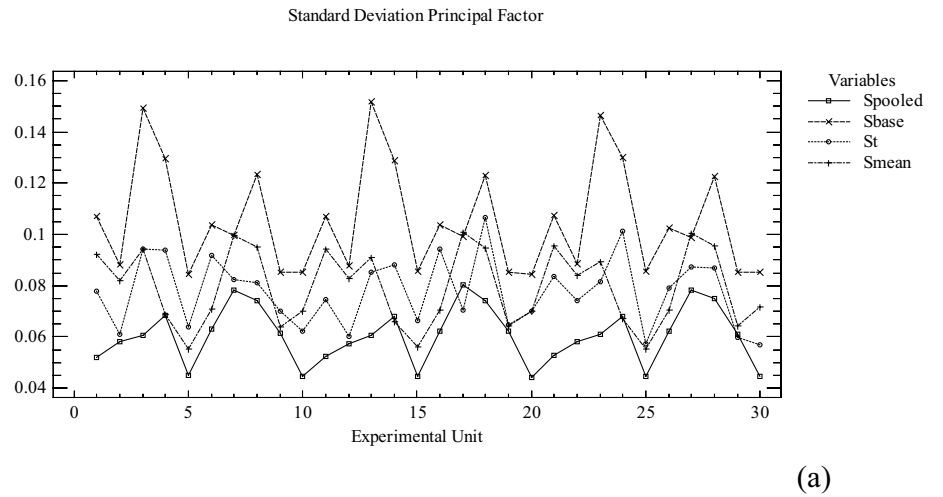
Figure 1 shows that the coefficient estimated through PF is more variable than that estimated through ML. This behavior is attributed to the fact that the variables were generated in a known normal multivariate distribution, which is a requirement for the estimation of maximum likelihood. All matrices used in the ML estimation, except for  $\mathbf{S}_{mean}$ , show an almost identical behavior. However, although  $\mathbf{S}_t$  presents similar results to  $\mathbf{S}_{pooled}$  and  $\mathbf{S}_{base}$ , comparisons are invalid when the matrix  $\mathbf{S}_t$  is used because it is not a fixed matrix.



**Figure 1: Coefficient for data simulated using Principal Factor (a) and Maximum Likelihood (b)**

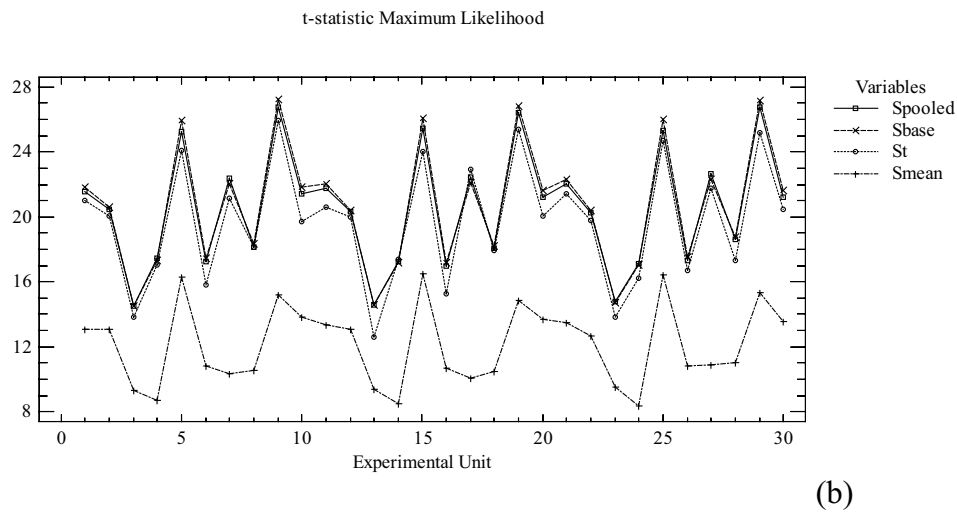
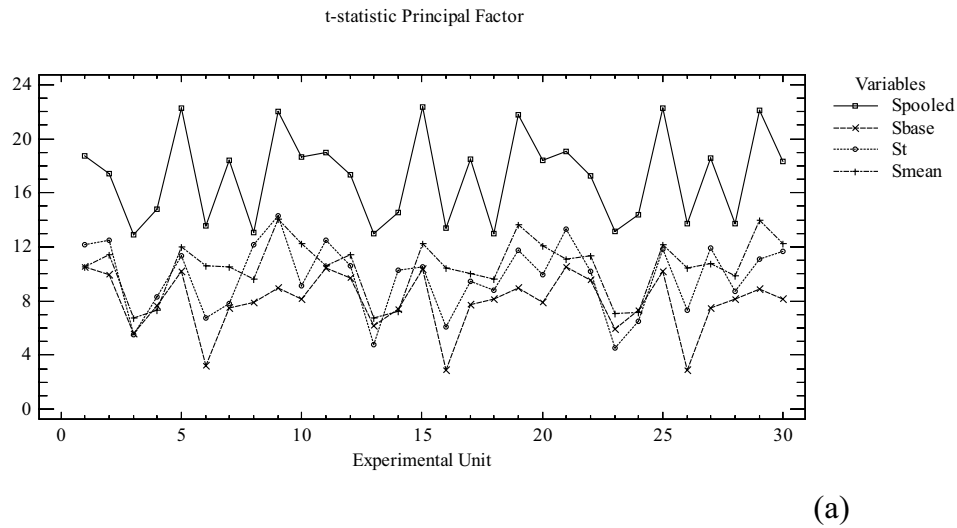
The standard deviations of the estimator, shown in Figure 2, account for the differences in both methods. Deviations obtained through PF are higher and show scale differences for all matrices;  $S_{base}$  and  $S_{pooled}$  follow the same tendency, but





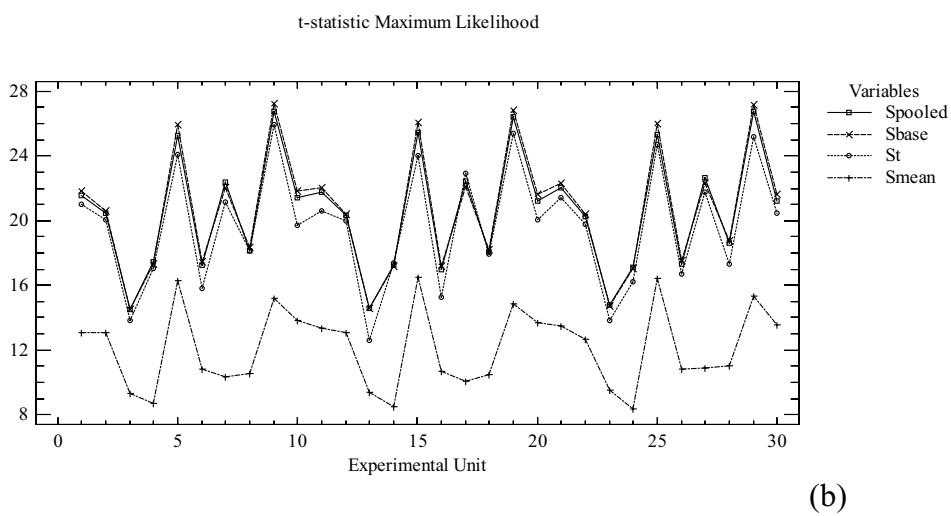
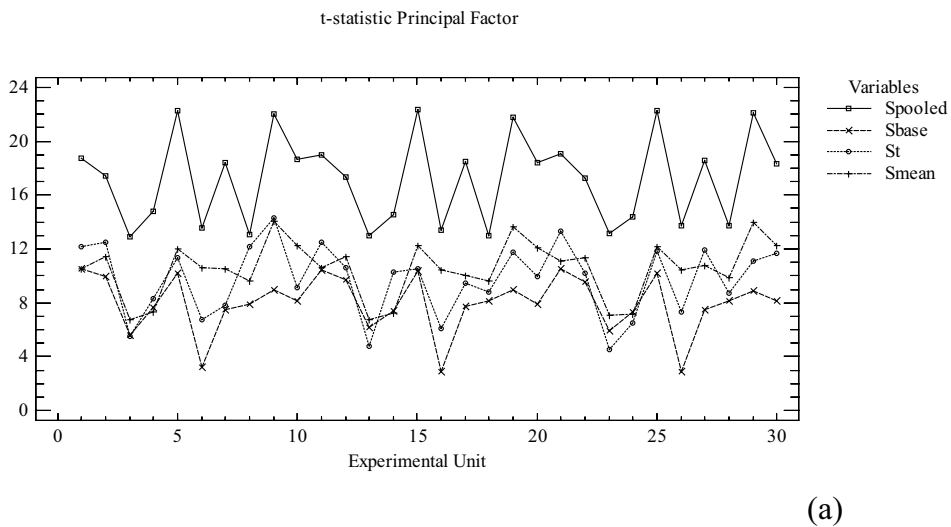
**Figure 2: Standard deviation for data simulated using Principal Factor (a) and Maximum Likelihood (b)**

$S_{mean}$  is closer to  $S_{base}$  even though they have opposite directions in some experimental units. Furthermore, only  $S_{mean}$  shows discrepancies in the results obtained through ML.



**Figure 3: Student's t-tests for data simulated using Principal Factor (a) and Maximum Likelihood (b)**

Figure 3 shows 'Student's t-tests estimates. For the PF method, matrix  $S_{pooled}$  is located above the others, resulting in highly significant t values, which is consistent with the increasing trend of the generated data; matrices  $S_{mean}$  and  $S_t$  are closer to  $S_{base}$ , which is located closer to t-values of 3 and 4, showing high significance. Nevertheless, the tendency of the  $S_{base}$  is equal to that of  $S_{pooled}$  for smaller values.



**Figure 4: Ranking for data simulated using Principal Factor (a) and Maximum Likelihood (b)**

The end result of measuring organizational performance is a ranking that classifies coefficients from high to low and p-values from low to high for the experimental units. Figure 4 shows the ranking for PF and ML. It was observed that the classification is very similar for both methods; matrix  $S_{mean}$  exhibits major differences from the others and matrix  $S_t$  has similar movements to  $S_{pooled}$  and  $S_{base}$ .

### 3.2 CASE STUDY

In general, organizations require performance measurement in order to control and assess the appropriate use of resources, which has become a financial matter of concern for industrial companies, leading to production issues. However, this practice has been extending to other organizations, including service providers such as higher education institutions, as illustrated in the studies below: Some countries in Europe (Benveniste, 1990) have proposed indicators for evaluating their higher education institutions; in Turkey (Celik and Ecer, 2009), Data Envelopment Analysis (DEA) is being used; in Australia (Horne and Hu, 2008), efficiency measurement is carried out using cost frontiers; among others. Latin American countries have also made these type of measurements, particularly in Colombia, where the performance of the State University System (SUE) has been measured since 2003, obtaining a series of capability and results indicators used to measure the 32 public universities of the country (Martínez et al. 2007). Results shown in the present article correspond to this case study.

The case study illustrates the performance measurement of the Colombian State University System (SUE), including 32 public university and 7 time periods from 2003 to 2009. The variables<sup>2</sup> used for this study were:

- Four (4) capability indicators (explanatory variables) namely Professors, Area in Square Meters, Budget and Spending on Administrative Staff, all weighted by the number of students.
- Four (4) results indicators (explained variables): Number of Indexed Journals, weighted by the Budget; Score by the Number of Articles Published in Indexed Journals, weighted by the Number of Educators; Number of graduates of Mater and PhD Programs, weighted by the Number of graduates; and Number of Professors under interchanges, weighted by the number of professors.

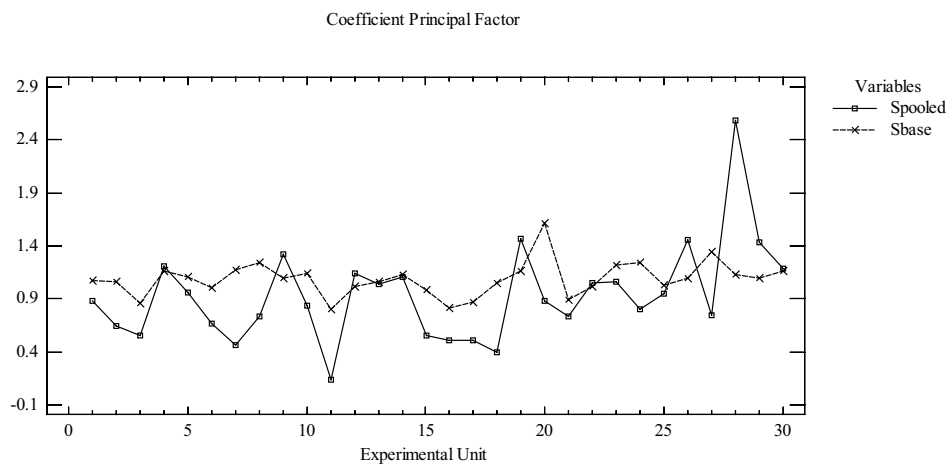
All variables showed asymmetry to the right, and thus they were transformed using a natural logarithm for the purpose of symmetrizing them and stabilizing the variance. Standardizing was also required giving the significant differences in the magnitudes or units of the variables.

As well as with simulated data, matrices were compared using the ratio of their determinants. Matrices  $S_{pooled}$  and  $S_{base}$  were used because they exhibited better simulation performance. Table 2 shows the results. It was observed that  $S_{pooled}$  has a lower concentration ellipsoid and, consequently, better estimates. Table 3 and Figures 5 to 8 show the results of applying the models.

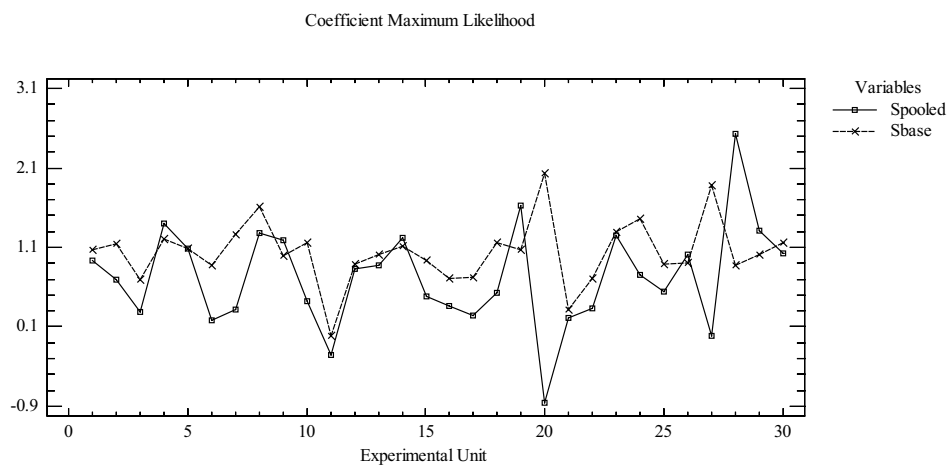
<sup>2</sup> The budget and spending on administrative staff were converted to constant pesos using 2003 as a base year; teaching hours were converted to equivalent full-time schedules

**Table 2: Ratio of determinants for the case study**

	Explanatory variables matrix		Explained variables matrix	
Matrix	$S_{pooled}$	$S_{base}$	$S_{pooled}$	$S_{base}$
Determinant	0.33	0.53	0.37	0.42
Ratio	1.00	0.63	1.00	0.88

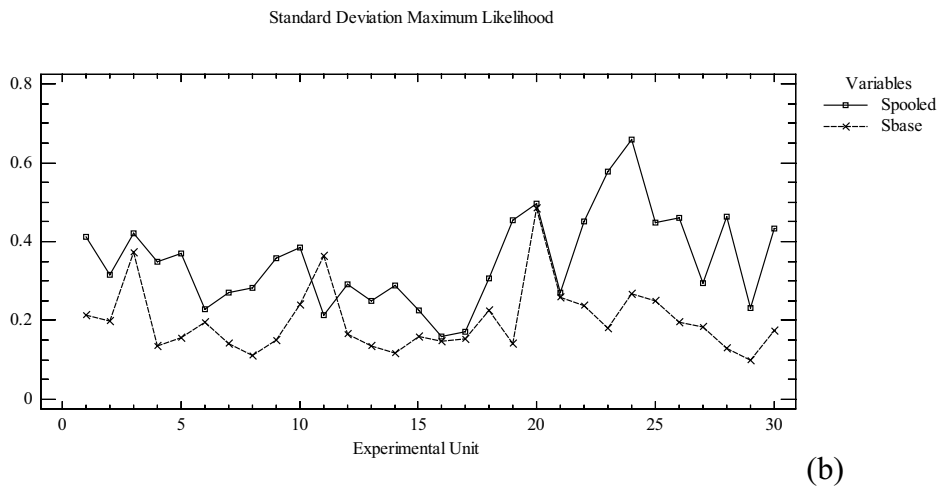
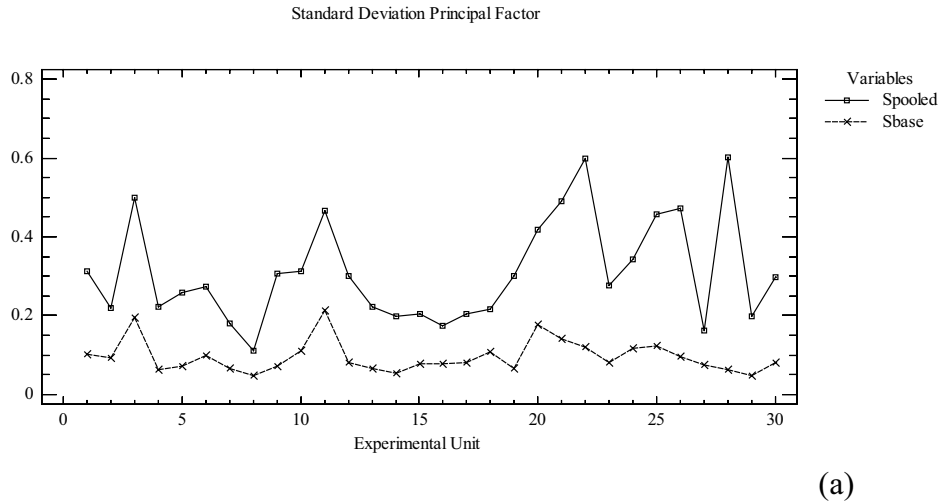


(a)



(b)

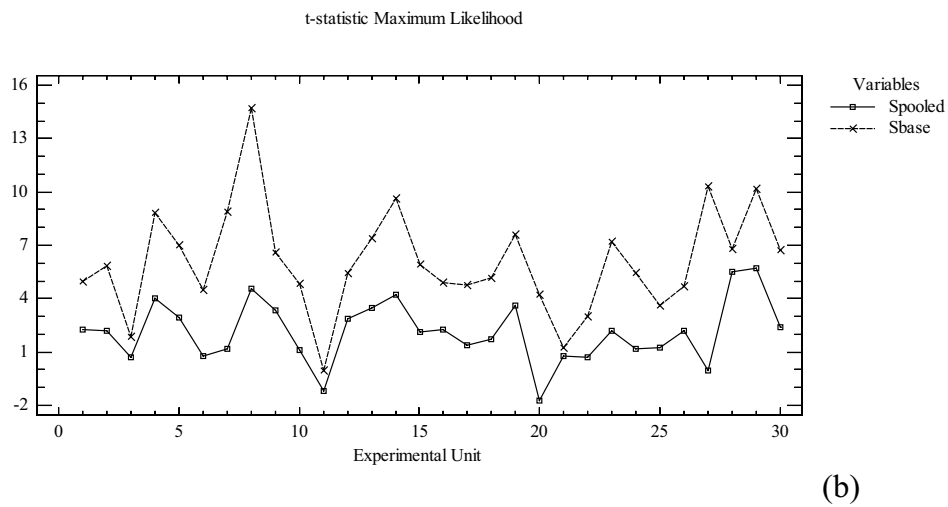
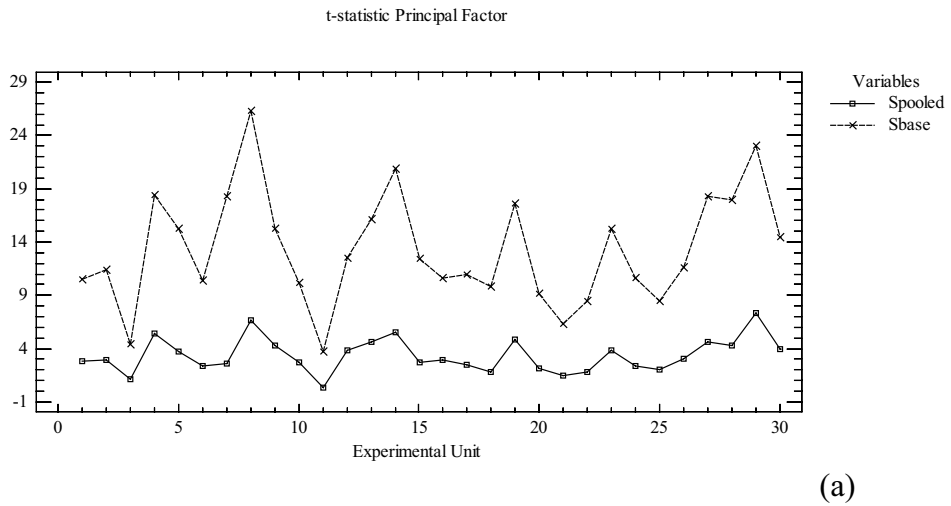
**Figure 5: Coefficient for the case study according to the Principal Factor (a) and Maximum Likelihood (b)**



**Figure 6: Standard deviation for the case study according to the Principal Factor (a) and Maximum Likelihood (b)**

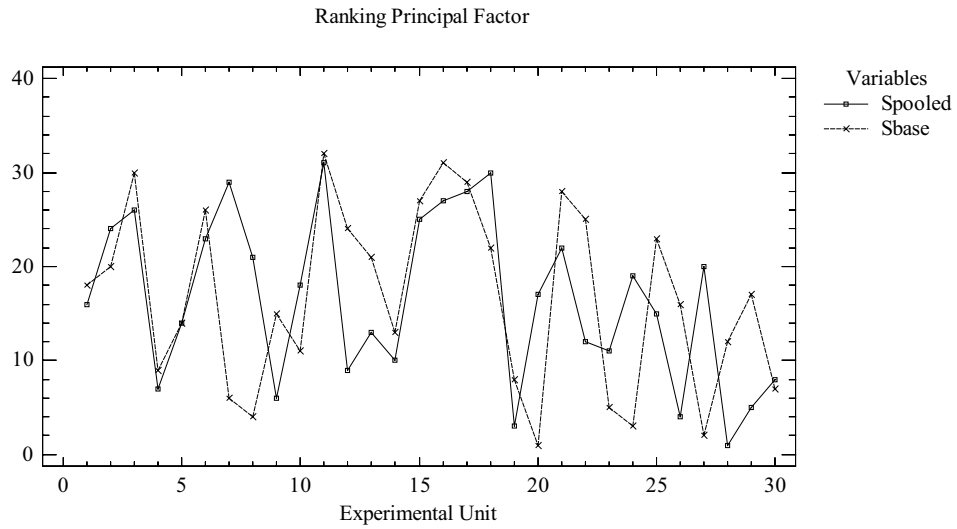
Figure 6 presents standard deviations of the estimator for the case study. PF shows a discrepancy among matrices,  $\mathbf{S}_{base}$  has less variability for both methods, and the matrices in ML have closer values.

Student's t-tests are shown in Figure 7, where it can be observed that  $\mathbf{S}_{base}$  values are above in the graph. Estimations obtained through PF using  $\mathbf{S}_{pooled}$

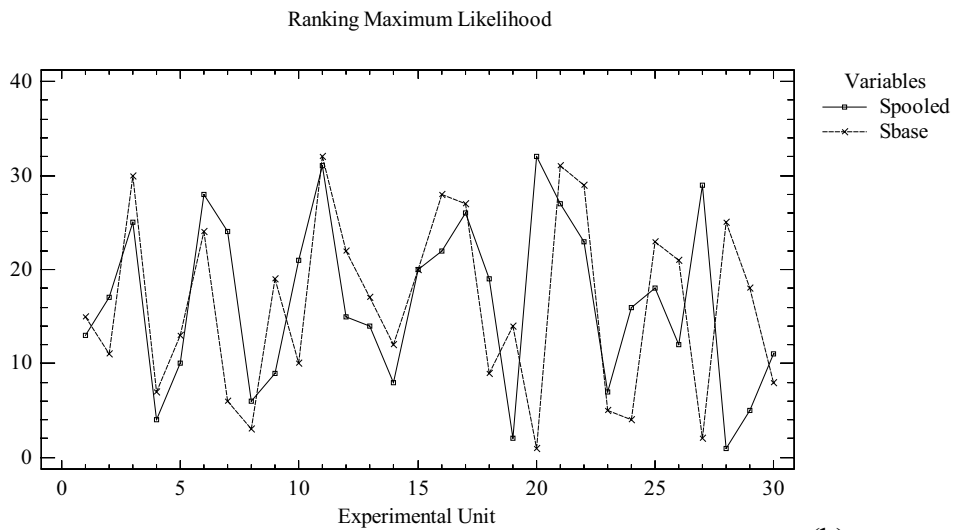


**Figure 7: Student's t-tests for the case study according to the Principal Factor (a) and Maximum Likelihood (b)**

showed to be more sensitive to changes in the universities, which will become evident in the significance of the p-value. Furthermore, when using ML,  $S_{base}$  will have a better performance in detecting these changes as Student's t-tests than  $S_{pooled}$  which has smaller values.



(a)



(b)

**Figure 8: Ranking for the case study according to the Principal Factor (a) and Maximum Likelihood (b)**

Figure 8 presents the ranking resulting from the application of panel data models. The location obtained through PF estimation shows a wider space than that obtained through ML. However, these locations are similar.



Table 3: Coefficient and p-values for 10 universities in the case study

Matrix	Estimation Method	University	$S_{pooled}$			$S_{base}$			$S_t$			$S_{mean}$		
			Coefficient	P-value	Coefficient	P-value	Coefficient	P-value	Coefficient	P-value	Coefficient	P-value	Coefficient	P-value
PF		1	0.89	0.01	1.07	0.00	0.50	0.18	0.79	0.02	0.02	0.00	0.79	0.02
		2	0.64	0.00	1.07	0.00	0.78	0.10	0.67	0.00	0.00	0.00	0.67	0.00
		3	0.55	0.27	0.85	0.00	0.15	0.73	0.48	0.35	0.00	0.00	0.48	0.35
		4	1.21	0.00	1.16	0.00	0.62	0.03	1.16	0.00	0.00	0.00	1.16	0.00
		5	0.96	0.00	1.11	0.00	0.05	0.83	0.72	0.01	0.00	0.00	0.72	0.01
		6	0.66	0.02	1.01	0.00	0.62	0.34	0.55	0.06	0.00	0.00	0.55	0.06
		7	0.46	0.01	1.18	0.00	0.47	0.07	0.27	0.14	0.00	0.00	0.27	0.14
		8	0.73	0.00	1.24	0.00	1.15	0.00	0.72	0.00	0.00	0.00	0.72	0.00
		9	1.32	0.00	1.10	0.00	0.51	0.11	1.11	0.00	0.00	0.00	1.11	0.00
		10	0.84	0.01	1.14	0.00	0.64	0.09	0.80	0.01	0.00	0.00	0.80	0.01
ML		1	0.94	0.02	1.06	0.00	0.79	0.13	0.91	0.07	0.07	0.13	0.91	0.07
		2	0.70	0.03	1.15	0.00	0.65	0.12	0.68	0.07	0.00	0.12	0.68	0.07
		3	0.29	0.50	0.69	0.07	-0.03	0.97	0.20	0.66	0.00	0.07	0.20	0.66
		4	1.40	0.00	1.20	0.00	1.41	0.04	1.38	0.00	0.00	0.04	1.38	0.00
		5	1.09	0.00	1.09	0.00	1.25	0.02	0.99	0.04	0.00	0.02	0.99	0.04
		6	0.17	0.45	0.88	0.00	0.01	0.98	0.07	0.76	0.00	0.00	0.07	0.76
		7	0.31	0.25	1.26	0.00	0.47	0.23	0.12	0.70	0.00	0.00	0.12	0.70
		8	1.28	0.00	1.62	0.00	0.66	0.00	0.98	0.01	0.00	0.00	0.98	0.01
		9	1.18	0.00	0.99	0.00	1.24	0.02	1.09	0.02	0.00	0.02	1.09	0.02
		10	0.43	0.27	1.16	0.00	0.42	0.45	0.20	0.62	0.00	0.00	0.20	0.62

#### 4. CONCLUSIONS

After performance measuring using panel data models based on factor analysis research was completed, the following conclusions were obtained:

- (1) The behavior of heterogeneous individuals (experimental units), both in temporal variation and in cross section, can be studied using panel data models, which makes them suitable for measuring organizational performance in situations in which variables are considered in a dichotomous manner, capability or results. It should be noted that the applicability of any statistical model depends on the nature of the phenomenon under study.
- (2) Factor Analysis, as a dimension reduction method, is appropriate for the problem under study. The aim of FA is to identify the true type of the latent variables generated by the observed variables, and scores can be used as an estimate of the underlying (latent) factors to be used in other statistical methods, such as panel data models. The Maximum Likelihood method shows a better performance due to the fact that the estimators obtained by PF are not either efficient nor invariant under linear transformations or changes in measurement units.
- (3) Outliers and influential data can have a stronger effect on  $\mathbf{S}_{pooled}$  since, by definition, this matrix is approximately located at the center of the  $\mathbf{S}_t$  matrices. This is also the case for  $\mathbf{S}_{mean}$ , generating systematic effects that affect the estimates. While  $\mathbf{S}_{base}$  cannot be affected by outliers and influential data due to the properties of index numbers, the base period must be properly chosen. In cases where the system under study actually shows an evolution, matrix  $\mathbf{S}_{base}$  should be considered.
- (4) When the study does not include either outliers and/or influential data, matrices  $\mathbf{S}_{pooled}$  and  $\mathbf{S}_{base}$  should be explored as they both have good estimates. The choice will be based on the analysis of information and the ratio of the determinants.
- (5) Although  $\mathbf{S}_t$  has estimates close to  $\mathbf{S}_{pooled}$ , which are more visible in the Maximum Likelihood method, it is not a fixed matrix, resulting in estimates that are not valid when assessing the temporal dynamics of experimental units.
- (6) Using matrices  $\mathbf{S}_{mean}$  and  $\mathbf{S}_t$  in the models presented herein is common practice; however, they are not supported by statistical theory and, thus, their conclusions are not valid. The first matrix has obvious discrepancies against the other matrices; the second matrix, as previously noted, is not a fixed matrix.
- (7) It is very important to note that, prior to the application of any statistical method, variables must be subjected to symmetrization through power transformation, which also contributes to stabilizing the series variance. The standardization of the transformed variables makes comparable variables with different units of measure.

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