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Pick the winner: an easy extension

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1. Introduction

In a recent paper with a very stimulating title: «An Exciting Alternative to Fisher's Exact Test for Two Proportions» published by Journal of Quality Technology, Carr (1985) faced the problem of selection of the better of two manufacturing methods with regard to the fraction nonconforming units. In his paper the Author, after addressing in a remarkably clear way the question of significance tests versus selection procedures, suggests the *Pick the Winner* approach. It consists simply in selecting a random sample of the same size from the set of units produced by each method and in picking up the method whose sample result is best. Statistical methodology contributes here in determining the sample size (n) such that there is a high probability (P) to pick up the *i-th* method ($i = 1,2$) whose sample has the lower number X_i of nonconforming units. Assuming that X_i is binomially distributed with mean p_i, the probability of correctly selecting method one as being better is:

$$P = \Sigma_{x=0}^{n} (Pr\ [X_1 = x])\ (Pr\ [X = x]\ /\ 2 + Pr\ [X_2 \geqslant x + 1])$$

where x is some particular value.

Alternatively one can solve the above problem by considering the usual approach of a test of significance for the comparison of two proportion and in addition to the risks α and β of first and second kind errors respectively, to take into account the risk (γ) of third error (Schwartz *et al.*, 1980). It relates to a serious kind of error implying, in this context, to conclude that the better manufacturing method is actually worse. According to the strategy discussed in the section «Selection Procedures» of Carr's paper if one puts $\alpha = 1.00$ $\beta = 0.00$, the $\gamma = 1\text{-}P$. The sample size must therefore be computed to ensure a *small value* of γ.

2. Some useful approximations

As far as two proportions are concerned, one can use the above reported formula (1) which allows the user to attain the exact sample size or can resort to the well known gaussian approximation:

$$n = \frac{z_\gamma^2}{2(\mathit{arcsen}\ \sqrt{p_1} - \mathrm{arcsen}\ \sqrt{p_2})^2}$$

where: p_1 = lower population fraction nonconforming
p_2 = higher population fraction nonconforming
z_γ = gaussian standardized deviate relative to the γ risk of third kind error.

If p_1, is less than or equal to 2% a better approximation (Chen, 1984) is given by:

$$n = \frac{z_\gamma^2}{2(\sqrt{p_1} - \sqrt{p_2})^2}$$

The rationale to get these equations is given by Schwartz *et al.* (1980).

Table I shows that the approximation to the exact values (in brackets) given by Carr is rather satisfactory.

3. Easy extension of the «Pick the Winner» procedure

The above approach enables one to easily extend the use of the *Pick the Winner* procedure to select the best of two means measured on a quantitative scale or the best of two treatments tested in terms of survival. The equation to determine the sample size in the first case is:

$$n = \frac{2\ z_\gamma^2\ \sigma^2}{d^2}$$

where: d = difference between two populations means which the researcher is interested in
σ^2 = population variance.

With regard to survival analysis, here the log-rank test is assumed to be

Tab. I – Sample size for γ = 10%, i.e. 90% confidence of a correct selection. Approximate values computed by expressions (2) or (3) (see text), and exact values (in brackets) given by Carr (1985).

	$\delta = 100p_2 - 100p_1$									
$100p_1$	1	2	3	4	5	6	7	8	9	10
0	164	82	55	41	33	27	23	21	18	16
	(160)	(80)	(53)	(39)	(31)	(26)	(22)	(19)	(17)	(15)
1	479	153	82	54	39	30	25	21	18	15
	(480)	(157)	(86)	(57)	(43)	(34)	(28)	(23)	(20)	(18)
2	793	239	122	77	54	41	33	27	23	20
	(798)	(236)	(121)	(77)	(55)	(42)	(34)	(28)	(24)	(21)
3	1105	311	153	94	65	48	38	31	25	22
	(1107)	(313)	(155)	(96)	(67)	(50)	(40)	(32)	(27)	(23)
4	1408	387	186	113	77	57	44	35	29	25
	(1410)	(389)	(188)	(115)	(79)	(58)	(46)	(37)	(31)	(26)
5	1705	460	219	131	89	65	50	40	33	28
	(1706)	(462)	(221)	(133)	(90)	(66)	(51)	(41)	(34)	(29)
6	1995	532	251	149	100	73	56	44	36	31
	(1995)	(534)	(252)	(150)	(101)	(74)	(57)	(45)	(37)	(32)
7	2278	603	282	166	111	80	61	49	40	33
	(2278)	(603)	(283)	(167)	(112)	(81)	(62)	(50)	(41)	(34)
8	2554	671	312	183	122	88	67	53	43	36
	(2554)	(672)	(313)	(184)	(123)	(89)	(68)	(54)	(44)	(37)
9	2824	738	342	200	132	95	72	57	46	38
	(2823)	(738)	(342)	(200)	(133)	(96)	(73)	(58)	(47)	(39)
10	3087	803	370	216	142	102	77	61	49	41
	(3086)	(803)	(371)	(216)	(143)	(103)	(78)	(61)	(50)	(41)
15	4304	1103	502	289	189	134	100	78	63	52
	(4302)	(1103)	(502)	(289)	(189)	(134)	(101)	(79)	(63)	(52)
20	5356	1363	616	352	229	161	120	93	74	61
	(5353)	(1362)	(616)	(352)	(229)	(161)	(120)	(93)	(75)	(61)
25	6244	1581	711	404	262	184	136	105	84	69
	(6240)	(1580)	(711)	(404)	(262)	(184)	(136)	(105)	(84)	(69)
30	6967	1757	788	447	288	201	149	115	91	74
	(6963)	(1756)	(787)	(447)	(288)	(202)	(149)	(115)	(91)	(75)
35	7526	1893	846	479	308	215	159	122	97	79
	(7521)	(1892)	(846)	(478)	(308)	(215)	(159)	(122)	(97)	(79)
40	7921	1988	886	500	321	224	165	126	100	81
	(7915)	(1986)	(886)	(500)	(321)	(223)	(165)	(126)	(100)	(81)
45	8151	2041	908	511	328	228	167	128	101	82
	(8145)	(2040)	(908)	(511)	(327)	(228)	(167)	(128)	(101)	(82)
50	8217	2053	912	513	328	227	167	127	100	81
	(8211)	(2052)	(911)	(512)	(327)	(227)	(167)	(127)	(100)	(81)

applied. In such case Freedman (1982) showed that sample size can be computed by a two steps approximate procedure. Firstly the pooled number of deaths (d) in the two samples is obtained by the following equation:

$$d = z_{\gamma}^{2} \left(\frac{1 + \vartheta}{1 - \vartheta} \right)^{2}$$

where: ϑ = constant of proportionality between the hazard functions of the two populations under investigation.

The size of the samples to be selected from each of the two populations is the computed by:

$$n = \frac{d}{2 - S_1\,(t^*) - S_2\,(t^*)}$$

when: $S_1(t^*)$ = % of surviving people in first population
$S_2(t^*)$ = % of surviving people in second population expected at time: $t = t^*$ of follow-up.

References

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